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PARAMAGNETIC RESONANCE IN METALLIC EUROPIUM AND INTERMETALLIC COMPOUNDS

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Kip[1] et al. have reported spin paramagnetic resonance in Gd metal which is due to the Gd ions in the $^8S_{7/2}$ state. Bozorth and Van Vleck[2] have recently concluded from magnetic susceptibility measurements that europium must also occur in the $^8S_{7/2}$ state. This implies, however, that europium, unlike the other rare earths, is divalent in its cubic body-centered form. If this assignment is correct and if the ionic spin relaxation time in europium is not excessive, then a paramagnetic resonance analogous to the one observed in gadolinium must be present. In a similar way paramagnetic resonance will be indicative of the valence of Eu in intermetallic compounds. In this Letter we report the result of resonance experiments in Eu metal and the compounds $EuIr_2$ and $GdIr_2$.

The experiments were carried out in the millimeter wave spectrometer described elsewhere.[3] This instrument uses no resonant cavity; instead, the europium was introduced as part of the waveguide wall, and the intermetallic compounds, which were only available as powders, were fastened to the walls with paraffin. Because of the complex geometry and a high standing-wave ratio, we cannot predict the exact absorption line shape, using Bloembergen's[4] theory; however, the line shape is still, for $\chi \ll 1$, a linear superposition of χ' and χ'', and since, for $\Delta H/H_0 \ll 1$, χ' and χ'' have simple symmetry properties around the resonance field H_0, we can determine this field to a fraction of the half-power linewidth ΔH.

Figure 1 shows the resonance line observed in europium metal. It occurs at $g = 1.985 \pm 0.015$. The linewidth $\Delta H = 1300$ gauss is approximately constant between room temperature and 120°K where it broadens and then disappears. The europium metal was produced by Research Chemicals and was said to contain no significant amount of paramagnetic impurities. The integrated strength of the line is approximately the same as the one found in gadolinium metal at 100°C. Efforts were made to keep the metal surface bright; however, if the surface was allowed to oxidize, the line did not change its appearance. Strength and its independence of surface state and the temperature behavior are evidence that this line is indeed due to europium metal.

The susceptibility measurements of Bozorth and Van Vleck find an anomaly (probably antiferromagnetic) in the europium susceptibility at 90°K. From the work of Maxwell and McGuire[5] it is known that line broadening sets in considerably above the transition point and, hence, we

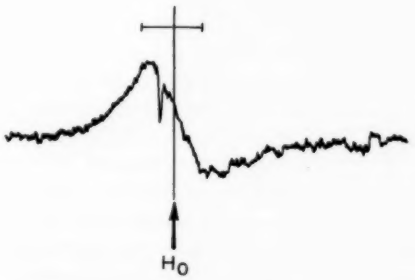

FIG. 1. Paramagnetic resonance of europium metal. Recording made at 51.0 kMc/sec. H_0 indicates the field for resonance.

attribute the temperature behavior of the linewidth to this transition.

The resonance line with a g factor of 1.985 cannot be explained from the behavior of Eu^{+++}, which is in the $4f^6$ configuration with the ground state $^7F_0$. The first excited state, $^7F_1$, which in the free ion is 480°K above the ground state, would give a resonance with a Landé g factor of $g = 9/8$. The paramagnetic resonance experiment confirms, therefore, the metallurgical[6] and magnetic[2] evidence for the divalent state of metallic europium. Below the antiferromagnetic anomaly (which is also borne out by electrical conductivity measurements on europium wire reported by Olsen[7]) the magnetic behavior is at present not well understood. Unlike the case of an antiferromagnet, the magnetization continues to increase. But we found no further magnetic resonance even at helium temperature. With the present apparatus, we could, however, not have seen a resonance with a g value smaller than 1.3. It is, therefore, not impossible that the resonance could have appeared at lower frequencies. If, on the other hand, this anomalous magnetic behavior were of similar origin to the one reported for cerium[8] (promotion of $4f$ electron into the $5d$ band), then observation of a resonance would be difficult because of the fast relaxation of the d electrons.

The intermetallic compound $EuIr_2$ crystallizes in the cubic Laves phase ($C15$) and from its lattice constant Zachariasen[9] concluded that Eu is here indeed in its trivalent state. Though it may become ferromagnetic, its saturation moment is almost vanishing.[10] In agreement with this evidence we have seen no paramagnetic resonance in this compound. $GdIr_2$, on the other hand, which is isomorphous with $EuIr_2$ and of course also trivalent, shows an absorption due to Gd which is 1500 gauss wide. This resonance increases in strength with decreasing temperature and has been followed to below the Curie point, where the resonance broadens to a point where observation is no longer feasible.

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[1]A. F. Kip, C. Kittel, A. M. Portis, R. Barton, and F. H. Spedding, Phys. Rev. 89, 518 (1953).
[2]R. M. Bozorth and J. H. Van Vleck, Phys. Rev. (to be published).
[3]M. Peter, Phys. Rev. 113, 801 (1959); J. B. Mock, Rev. Sci. Instr. (to be published).
[4]N. Bloembergen, J. Appl. Phys. 23, 1383 (1952).
[5]L. R. Maxwell and T. R. McGuire, Revs. Modern Phys. 25, 279 (1953).
[6]F. H. Spedding, A. H. Daane, and K. W. Herrmann, Acta Cryst. 9, 559 (1956); W. Klemm and H. Bommer, Z. anorg. u. allgem. Chem. 231, 138 (1937).
[7]C. E. Olsen, Los Alamos Scientific Laboratory Report LASL-2406 (unpublished).
[8]T. Murao and T. Matsubara, Progr. Theoret. Phys. (Kyoto) 18, 215 (1957).
[9]W. H. Zachariasen (private communication).
[10]R. M. Bozorth, B. T. Matthias, H. Suhl, E. Corenzwit, and D. D. Davis, Phys. Rev. 115, 1595 (1959).

DESTRUCTION OF DIATOMIC BONDS BY PRESSURE*

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The pressure necessary to destroy the directional character of chemical bonds has been discussed theoretically in connection with hydrogen.[1,2] This phenomenon has now been experimentally observed in iodine as a first-order phase transition, since the delocalization of the electrons causes the diatomic molecular crystal to be converted into a monatomic metal. The results for iodine can be used to evaluate the predictions made for hydrogen.

The experiments were carried out by standard single-shock experiments[3] on polycrystalline iodine at an initial density within 1% of the single-crystal density. The measured shock velocity, u_S, versus particle velocity, u_p, is shown in Fig. 1. The experiments again conform within the accuracy of the data to the empirical linear relation between these two quantities. Moreover, the line extrapolates properly to the experimental sound speed (1.35 mm/μsec) at zero particle velocity. The square points are from slightly less accurate experiments. The

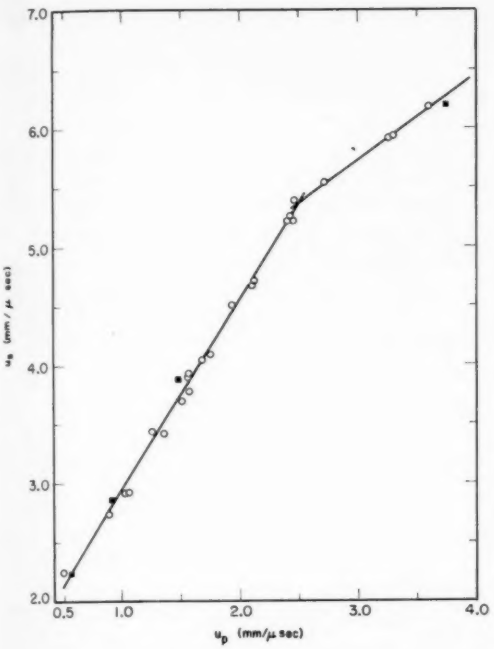

FIG. 1. The shock velocity, u_S, versus the particle velocity, u_p, in iodine.

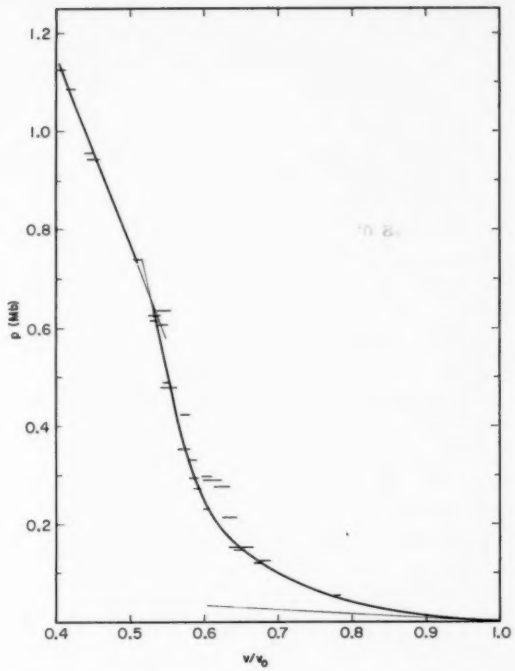

FIG. 2. The Hugoniot equation of state of iodine.

intersection between the two linear segments is used to locate the transition. From u_S and u_p the Hugoniot equation of state is calculated, with the result as shown in Fig. 2. The pressure, p, is in megabars and v/v_0 is the volume relative to the volume at 1 atmosphere, v_0. The length of the horizontal lines indicates the probable error of each experiment. In calculating the probable error no account has been taken of variation in initial density. The early experiments near 0.3 megabar had a low initial density and hence were not heavily weighted in drawing the curve. The lightly drawn parts of the curve are extrapolations near the transition which occurs approximately at a pressure of 0.7 megabar and at a relative volume of 0.53.

Upon converting the Hugoniot curve to an isothermal equation of state using the assumptions of a constant heat capacity, $c_v = 6Nk$, and a Grüneisen constant, $\gamma = 2.07$, in connection with the theory of reference 3, it is found that at the transition point the temperature is almost 1 ev and that the room temperature transition pressure would be approximately 0.3 megabar at $v/v_0 = 0.53$. This does not mean that upon hydrostatic compression iodine would necessarily have the phase transition at that pressure, since the temperature dependence and the rate of transition of the phase change have to be considered.

The temperature of about 1 ev is comparable to the dissociation energy of the iodine molecule (1.53 ev) and hence could have a considerable effect in reducing the pressure of the phase change. By further work using double-shock techniques, the temperature at a given pressure can be considerably lowered and the temperature dependence of the phase transition can be established. As for the rate of transformation, these dynamic experiments require that the phase change take place in less then 1 μsec, namely within the time the iodine is subjected to the pressure. Thus, in order to see any transformation at all the system may have to be overdriven to the pressure and temperature conditions where it will occur that fast. Previous experience[4] has indicated that these simpler phase changes in solids can take place that rapidly without overdriving. Static isothermal experiments, on the other hand, may also have to be overdriven in order for the rate of transformation to be observable in a finite time.

To make sure that the observed phase change

is the one postulated, some physical property of the new phase would have to be measured. The obvious one under shock conditions might be the conductivity; however, the conductivity might be metallic before the existence of a monatomic lattice if the conduction band overlaps the valence band in the diatomic system. This change might be difficult to detect in the Hugoniot equation of state. Although conductivity measurements have been made at low pressures to show the narrowing down of the energy gap, the present experimental techniques in any case do not allow the determination of very low resistances existing for such short times.

However, an empirical rule for the lengthening of diatomic bonds to metallic bonds can be used to calculate at what volume iodine should have its transition and thus confirm the suggestion made. The empirical evidence can be obtained from the alkali metals which have a diatomic species in the vapor phase and condense into a body-centered cubic metal lattice. From the known interatomic distances[5] it is found that the Li spacing expands by 13.7%, Na by 20.7%, and K and Cs both expand by 15.8%. From the known diatomic distance in iodine and the assumed expansion of 15.8% in interatomic spacing for the heavy elements and also assuming a body-centered structure for the metallic iodine phase, the phase transition for iodine would be predicted to occur at $v/v_0 = 0.53$. This is in agreement with the experimental value. Other elements for which this expansion in spacing is known all fall into the 10 to 20% region.

Earlier theoretical calculations on metallic hydrogen[2] have predicted this expansion of the bond to be about 80%. From quantum mechanical calculations on the metal-like triatomic hydrogen system[6] it can be made plausible that here also the expansion should be near 20%. A 20% expansion over the diatomic distance would mean that, for the monatomic phase of hydrogen to exist, the normal solid phase would have to be compressed by a factor of 30 instead of 10 as previously predicted. The pressure of this transition is then calculated by the Wigner method[1] to be near 20 megabars instead of the 1 megabar estimated earlier.

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[1]E. Wigner and H. B. Huntington, J. Chem. Phys. 3, 764 (1935).

[2]R. Kronig, J. de Boer, and J. Korringa, Physica 12, 245 (1946).

[3]J. M. Walsh, M. H. Rice, R. G. McQueen, and F. L. Yager, Phys. Rev. 108, 196 (1957).

[4]R. E. Duff and F. S. Minshall, Phys. Rev. 108, 1207 (1957).

[5]Tables of Interatomic Distances and Configuration in Molecules and Ions (The Chemical Society, London, 1958).

[6]G. E. Kimball and J. G. Trulio, J. Chem. Phys. 28, 493 (1958).

NEUTRON SMALL-ANGLE SCATTERING BY SPIN WAVES IN IRON

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Neutron small-angle scattering by spin waves in iron has been observed in virtual isolation at an intensity of several thousand counts per minute, the various other effects that normally complicate such a measurement having been rendered negligible. The theoretical dependence of the scattered intensity on neutron wavelength and scattering angle and on specimen temperature, setting, and magnetization has been confirmed. If iron be regarded as a Heisenberg-Bloch ferromagnet, the cutoff angle of the scattering gives a value of 0.018 ev for the effective exchange integral, in agreement with the value obtained by fitting a $T^{3/2}$ law to the saturation magnetization at low temperatures.

Although it is well known that neutron scattering techniques provide a sensitive means for the detection of spin waves, experiments have been performed on three substances only – magnetite,[1] hematite,[2] and iron.[3] A source of complication is that the spin-wave scattering associated with any planes of nonzero *hkl* in a magnetic crystal is accompanied by phonon scattering whose response to changes in magnetization and temperature must be allowed for. In the present experiment we have avoided this difficulty by studying the scattering at positions close to the main beam, where 1-magnon inelastic scattering is

allowed, but where there is no 1-phonon scattering of neutrons having velocity less than that of sound in the crystal (i.e., in iron, of neutrons with wavelength >1.1 A).

The expected small-angle scattering may be calculated from formula (2.37) of reference 4. For iron it has the distribution shown in Fig. 1. This distribution has two remarkable features: its intensity – the weakest point corresponds at room temperature to a differential cross section of $35\lambda^2$ barns steradian^{-1} atom^{-1} (λ in A); and the fact that to a high degree of approximation it is independent of incident wavelength. Exploiting the latter feature, we allowed a ribbon-shaped beam of Maxwell neutrons from DIDO, 1.60 cm high, 0.125 cm wide, and with cadmium ratio ~1900, to fall upon a cylindrical iron crystal of height 2.14 cm and diameter 0.739 cm. The reactor temperature, specimen thickness, and counter efficiency were such that 92% of the recorded spin-wave-scattered neutrons were, on incidence, slower than the velocity of sound in iron. A vertical counter slit of width 0.190 in. could be tracked across the beam at a distance of 57.3 in., picking up the intensity between parallel lines in Fig. 1; as the incident collimation was better than 6′ of arc the small-angle diffraction pattern could be surveyed down to angles of ~15′. By rotating an applied magnetic field, the expected spin-wave pattern could be turned around the optical axis so as to move the slit effectively from H to V.

We found the following results:

1. Intensity having the anticipated angular dependence was present. Figure 2 shows the difference between the counting rates at H and V as a function of scattering angle and temperature. A form of magnetization-dependent, temperature-dependent scattering is there revealed that cuts off sharply at (correcting for instrumental widths) $25 \pm 1'$, in exact agreement with the theoretical 25′.

2. On rotating the magnetic field from H to V, the expected curve of intensity variation was obtained. At 25′ the intensity ratio H/V was 1.46 $\pm$ 0.02, as against a theoretical 1.40.

3. Absolute intensities were below those calculated for a simple Heisenberg-Bloch ferromagnet, as they must be for a real crystal with anisotropic interactions.[4] For instance, at 25′ the H counting rate (above a background and disorder-scattering contribution of 240 ± 3 min^{-1})

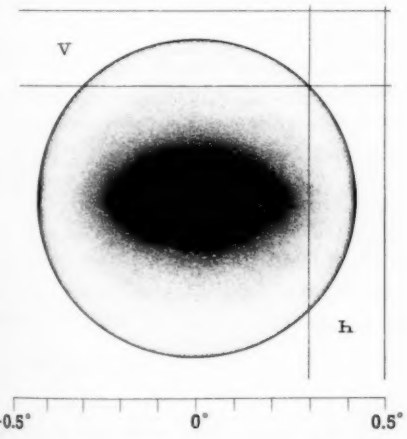

FIG. 1. The theoretical distribution of neutrons scattered near the primary beam by spin waves in iron, assuming perfect collimation. Parallel lines show the effective position in this pattern of the counter slits for horizontal (H) and vertical (V) applied magnetic field.

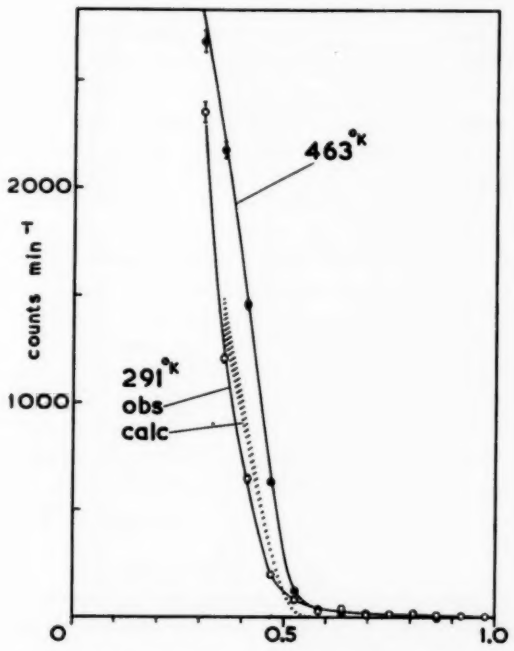

FIG. 2. Difference between the observed counting rates at H and V as a function of angle and temperature, showing a sharp cutoff at the angle predicted. The calculation is for a simple Heisenberg-Bloch ferromagnet without anisotropic interactions.

was 2000 ± 15 min^{-1}, as against a theoretical 3450 ± 240. Crystal-defect scattering was present at some specimen orientations, and was investigated with results that will be described in a more detailed communication.

4. An absorption curve in silver showed that if the differential cross section giving rise to the intensity has the form λ^n, then $n = 2.0 \pm 0.1$. Theory requires λ^2.

5. For some 54°K above room temperature the intensity was proportional to the absolute temperature, and the temperature increase was independent of the crystal setting.

The above points are decisive in identifying single-magnon scattering. An unexpected feature, observed with three different specimens, is that as the temperature of the specimen is raised a markedly nonlinear increase of intensity sets in abruptly at ~346°K (Fig. 3). This apparently sudden failure of the single-spin-wave discussion at a well-defined temperature is strongly reminiscent of the sudden deviation from a $T^{3/2}$ law found by Foner and Thompson[5] in the saturation magnetization of nickel. It is possible to argue from our data that the nonlinear intensity is unconnected with phonons, multiple-quantum scattering, multiple scattering, or crystal defects, nor is it explained by a temperature variation of J_{eff}. Thus it would appear that a moderately excited spin system possesses states that have not hitherto been adequately discussed. The confinement of the nonlinear intensity within 25′ of angle seems to indicate that spin-wave states continue to be involved, and is inconsistent[6] with any application to iron of Wohlfarth's suggestion[7] that collective-electron orbitals may come into play at the temperature where spin-wave theory breaks down for nickel.

We are grateful to Dr. R. J. Elliott for a valuable discussion, to Mr. E. B. Fossey for a computation and to the Monocrystals Company for the iron crystals.

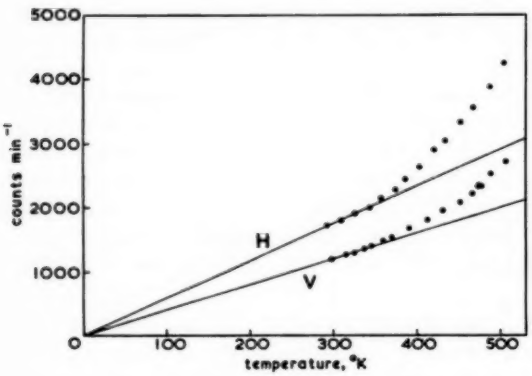

FIG. 3. Temperature dependence of the intensity at a scattering angle of 25′.

*On attachment from the Atomic Energy Establishment, Trombay, India.

[1]T. Riste, K. Blinowski, and J. Janik, J. Phys. Chem. Solids 9, 153 (1959); B. N. Brockhouse, Phys. Rev. 111, 1273 (1958).

[2]J. A. Goedkoop and T. Riste, Nature 185, 450 (1960).

[3]R. D. Lowde, Proc. Roy. Soc. (London) A235, 305 (1956).

[4]R. J. Elliott and R. D. Lowde, Proc. Roy. Soc. (London) A230, 46 (1955).

[5]S. Foner and E. D. Thompson, Suppl. J. Appl. Phys. 30, 229S (1959).

[6]R. J. Elliott, Proc. Roy. Soc. (London) A235, 289 (1956).

[7]E. P. Wohlfarth and J. F. Cornwell, Berichte der Arbeitsgemeinschaft Ferromagnetismus 1959, p. 9 (1960).

OPTICAL ABSORPTION BY DEGENERATE GERMANIUM

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In a previous note[1] the author has proposed that degeneracy due to impurities in germanium results in an effective shrinkage of the energy gap. This hypothesis had been suggested by the threshold of the emission spectrum which puts the conduction band edge at about 0.5 ev from the valence band. In this case the impurity concentration was estimated as being of the order of 10^{19} cm^{-3}.

Optical transmission measurements were made on wafers of germanium doped with a known concentration of arsenic. Figure 1 shows the optical density of two heavily doped specimens and that of a pure specimen for comparison. Because of free carrier absorption it is difficult to identify the threshold corresponding to the

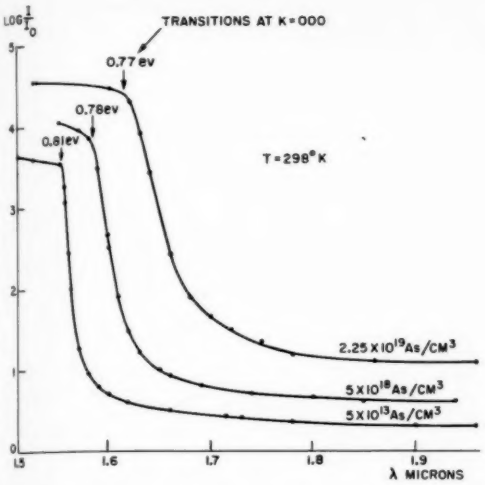

FIG. 1. Optical density of arsenic-doped germanium.

band edge at $\vec{k}=(1,1,1)$. However, the data clearly show how doping moves the (0, 0, 0) valley of the conduction band to a lower energy.

A judicious plot of the transmission data can separate the effect of free carrier absorption. Let $I_0(\lambda)$ be the radiation incident on the specimen and assume that the reflection coefficient R is the same for all specimens; then $I_p(\lambda)$, the radiation transmitted by the pure specimen, is

$$I_p(\lambda)=I_0(\lambda)(1-R)^2 e^{-\alpha(\lambda)x},$$

where $\alpha(\lambda)$ is the absorption coefficient and x is the thickness of the wafer. Note that, since there are very few free carriers, $\alpha(\lambda)$ is purely due to edge absorption. The radiation transmitted by a doped specimen is

$$I_d(\lambda)=I_0(\lambda)(1-R)^2 e^{-[\alpha'(\lambda)+\alpha_f(\lambda)]x},$$

where $\alpha'(\lambda)\neq\alpha(\lambda)$, and $\alpha_f(\lambda)$ is the free carrier absorption. Then

$$\ln[I_p(\lambda)/I_d(\lambda)]=[\alpha'(\lambda)-\alpha(\lambda)+\alpha_f(\lambda)]x.$$

$\alpha_f(\lambda)=a\lambda^n$ is dominant at long wavelength whereas α and α' are effective in the vicinity of 2 μ. For free carriers in metals $n=2$, but for germanium values of n up to 3.5 have been reported.[2,3]

In Fig. 2 we have plotted $\log\{\log[I_p(\lambda)/I_d(\lambda)]\}$ vs $\log\lambda$. The data show that free electron absorption in the range studied follows the λ^2 law.

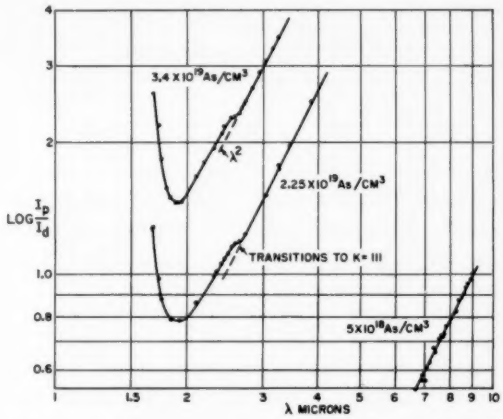

FIG. 2. Increase in absorbance due to high concentrations of donors in germanium.

If the departure from this law at short wavelengths is attributed to the onset of edge absorption, we are led to conclude that the edge of the degenerate conduction band is less than 0.48 ev above the valence band (allowing for the fact that no edge absorption can take place to those states of the conduction band which are already occupied[4]).

The data of Fig. 1 were obtained with a Cary double-beam spectrometer using quartz optics. The sensitivity beyond 2 μ was too poor to reveal the onset of edge absorption. Therefore, another instrument (Perkin Elmer Model 112) with NaCl prism was used to explore the spectral range of Fig. 2 not covered by the first instrument.

The evidence then indicates that for germanium, unlike the case for InSb,[4,5] the shrinkage of the energy gap is greater than the rise of the Fermi level.

A series of measurements of absorption as a function of doping and at low temperature is under way.

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[1]J. I. Pankove, Phys. Rev. Letters 4, 20 (1960).
[2]H. Y. Fan and M. Becker, Semiconducting Materials (Butterworths Scientific Publications, Ltd., London, 1951), p. 132.
[3]H. Y. Fan, W. Spitzer, and R. J. Collins, Phys. Rev. 101, 566 (1956).
[4]E. Burstein, Phys. Rev. 93, 632 (1954).
[5]P. Aigrain and J. des Cloizeaux, Compt. rend. 241, 859 (1955).

DIRECT OBSERVATION OF POLARONS AND PHONONS DURING TUNNELING IN GROUP 3-5 SEMICONDUCTOR JUNCTIONS

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Narrow p-n junctions in which most of the conduction for small applied voltages results from tunneling have been studied in the 3-5 compounds InSb, InAs, InP, GaSb, GaAs, and GaP. With the exception of GaP, all of these compounds are believed to have both conduction and valence band edges located at the center of the Brillouin zone. Thus, the tunneling of an electron from the bottom of the conduction band to the top of the valence band does not necessitate a large change of wave vector as in Ge and Si tunnel diodes[1] and the observed structure in the electrical characteristics produced by the assisting phonons is absent. Instead, the low-temperature electrical characteristics of group 3-5 junctions show structure of an entirely different kind which is clearly related to the polar character of these semiconductors and whose magnitude correlates with the polar electron-phonon coupling constant[2] $\alpha = (e^2/\hbar)(m^*/2\hbar\omega_l)^{1/2}(\epsilon_\infty^{-1} - \epsilon_0^{-1})$. Threshold voltages are observed corresponding to the energy $\hbar\omega_l$ necessary for the creation of a long-wavelength longitudinal optical phonon. A pronounced conductance minimum, which appears near zero bias, is attributed to a threshold at the energy required to move an electron adiabatically from a polarized electron state to a polarized hole state.

The junctions were prepared by fusing mixtures of suitable alloys to degenerately doped n- or p-type semiconductor wafers. The alloys which were used consisted of elements from the second, fourth, or sixth columns of the periodic table and sometimes contained further additions of the Column 3 constituent of the wafer, i.e., Ga or In.

The electrical characteristics were observed by cooling the diodes to 4.2°K by immersion in liquid helium and plotting the current-voltage characteristic using an X-Y recorder. While the structure to be described can be seen by careful inspection of the current-voltage characteristic itself, it is more clearly revealed by a circuit (suggested by J. J. Tiemann) which causes the recorder to plot the conductance (dI/dV) as a function of the voltage, V. Photographs of several representative recordings of this kind are shown in Fig. 1. The data which we report were

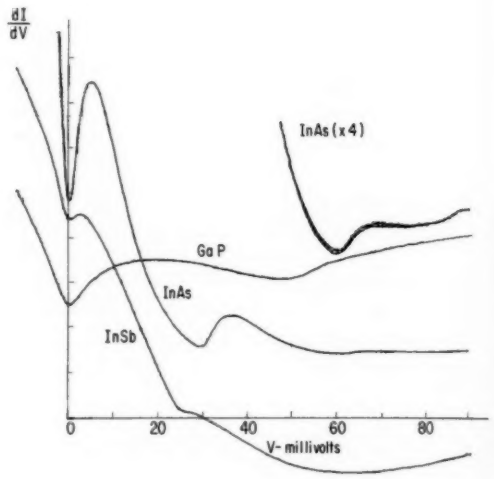

FIG. 1. Conductance (dI/dV) in arbitrary units vs voltage for several group 3-5 junctions.

obtained from measurements of at least four junctions in each of the materials studied, with the exception of InP where only one junction was examined.

The conductance curves exhibit distinct minima in the neighborhood of zero voltage. These minima are invariably observed in these polar semiconductors and they tend to be more pronounced in compounds having a larger electron-phonon interaction. We interpret the reduced transition probability near zero bias as arising from the lowering of the electron and hole energies due to self-interaction with the phonon field (polaron formation). In tunneling processes that occur rapidly compared to the lattice relaxation time and at sufficiently low temperatures that the Fermi distribution characterizing the electrons is sharp, the applied voltage must supply the relaxation energy necessary to move the electron rapidly between differently polarized regions of the crystal. It can be shown by consideration of the energy cycle for the process that this relaxation energy is $\frac{1}{2}(\alpha_c + \alpha_v)\hbar\omega_l \approx \alpha_c\hbar\omega_l$, where α_c and α_v are the coupling constants for the conduction and valence bands, respectively. The thresh-

Table I. (Energies in ev $\times 10^{-3}$.)

	Deduced from other expts.			Present expts.	
	α_c	$\hbar\omega_l$	$\alpha_c\hbar\omega_l$	$\alpha_c\hbar\omega_l$	V_t
InSb	0.02	22.8	0.4	0.4 ± 0.1	24
InAs	0.05	28.8	1	1 ± 0.5	30.5
InP	0.08	42.0	4	4	45
GaSb	0.02	29.0	0.6	0.5 ± 0.2	29.6
GaAs	0.06	35.2	2	0.1 ± 0.05	36
GaP	0.1	49.9	6	6 ± 3	52

old voltages for this process are not clearly resolved in Fig. 1 because of temperature broadening which, at 4.2°K, is sufficient to obliterate much of the detail which this model suggests.

The rough comparison, shown in Table I, of the threshold estimated from the magnitude of the minimum in the experimental curves with the self-energy calculated from independent experimental data (but not corrected for screening[3]) lends credence to the present hypothesis. The agreement between observed and calculated values is reasonable, except in the case of the GaAs junctions, which were narrower and much more heavily doped than the others.

The possibility that the conductance minimum is associated with the creation of low-energy acoustical phonons can be ruled out by the absence of a corresponding temperature dependence of the tunneling current. The fact that the valence band edges are not at $k = 0$ but slightly displaced[4] is also unable to account for this effect, since estimates show that it is too small, and in addition the correlation with the spin-orbit splitting to be expected under these conditions is absent.

It is seen from Fig. 1 that the slope of the conductance curve again changes abruptly at a higher voltage V_t. We interpret this phenomenon as the threshold for a new process which is characteristic of the host crystal and demonstrably independent of the concentration or chemical nature of the impurities used to produce the junction. It represents the onset of tunneling processes in which a longitudinal optical phonon is emitted simultaneously with the passage of an electron through the junction. As shown in Table I, the observed threshold voltages agree within experimental error with the sum of the polaron relaxation energy and that of the zone-center longitudinal optical phonon as calculated from optical reflectivity data.[5,6] Since these processes depend upon the coupling of the electron to the lattice, their probability may be expected to be proportional to α. This expectation is qualitatively confirmed by the observation that the structure is more pronounced in the more polar compounds, InAs, InP, and GaP, and is relatively weak in junctions made from InSb, GaSb, and GaAs, where the coupling constant is smaller.

The conductance curve for InAs, shown in Fig. 1, exhibits an additional contribution to the tunneling current beyond a voltage which is equal to $(2+\alpha_c)\hbar\omega_l$ within the accuracy of these experiments. This component, which presumably corresponds to the tunneling of an electron accompanied by two optical phonons, is frequently observed in the characteristics of the more polar 3-5 compounds, with an amplitude that is smaller than that of the one-phonon process by a factor of 5 or 10.

The fact that structure corresponding to that just discussed is totally absent in Ge and Si is due to the considerably weaker electron-phonon coupling in these materials as contrasted to the rather strong interaction in the group 3-5 semiconductors arising specifically from their polar character.[3]

The type of measurement discussed here permits a direct experimental determination of the polar electron-phonon coupling constant. Furthermore, these results imply that the tunneling process takes place in a time short compared with the lattice relaxation time.

We are grateful to N. Holonyak for supplying some of the InP and GaP junctions, and to H. Haken and H. Brooks for helpful discussions.

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[1]H. Holonyak, Jr., I. A. Lesk, R. N. Hall, J. J. Tiemann, and H. Ehrenreich, Phys. Rev. Letters 3, 167 (1959).

[2]H. Fröhlich, Advances in Physics, edited by N. F. Mott (Taylor and Francis, Ltd., London, 1954), Vol. 3, p. 325.

[3]H. Ehrenreich, J. Phys. Chem. Solids 2, 131 (1957); 8, 130 (1959).

[4]E. O. Kane, J. Phys. Chem. Solids 1, 245 (1957).

[5]G. Picus, F. Burstein, B. W. Henvis, and M. Hass, J. Phys. Chem. Solids 8, 282 (1959).

[6]D. Kleinman and W. Spitzer, Phys. Rev. 118, 110 (1960).

NUCLEAR SPIN RELAXATION IN SOLID He^3†

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The liquid states of both helium isotopes have been of particular interest due to their peculiar macroscopic properties which have been attributed to quantum effects. In solid He^3 the nuclear susceptibility departs from Curie's law,[1-3] below 0.2°K, indicating that some exchange interaction must be present in addition to the direct dipole-dipole interaction between atoms. Thus solid He^3 is also found to exhibit some such peculiar properties. A further distinctive property of solid He^3 is the α-β phase transition discovered by Grilly and Mills.[4] This transition has no analog in solid He^4, and its nature is not yet understood. Some relevant information contained in the original work of Grilly and Mills is that the volume change associated with the transition approaches zero at approximately 2°K. In the work to be described here, nuclear spin relaxation times were measured in solid He^3 such that they might yield additional information as to these properties.

Free-precession techniques were used to measure both the transverse and longitudinal relaxation times at 30.4 Mc/sec to an accuracy of roughly 10%. The measurements were made both as a function of pressure at constant temperature and as a function of temperature at constant volume. In order to allow pressure variation of the sample, a loop of resistance wire was placed inside of the capillary through which the He^3 entered the sample chamber. In this way, by passing a current pulse through the wire, the plug of solid He^3 which formed in the capillary could be melted momentarily and the solid in the sample chamber could come to pressure equilibrium with the vapor in the warmer part of the capillary.

Large discontinuities in both relaxation times were observed as the pressure was increased or decreased across the α-β phase boundary above approximately 2°K. Figure 1 shows the data at

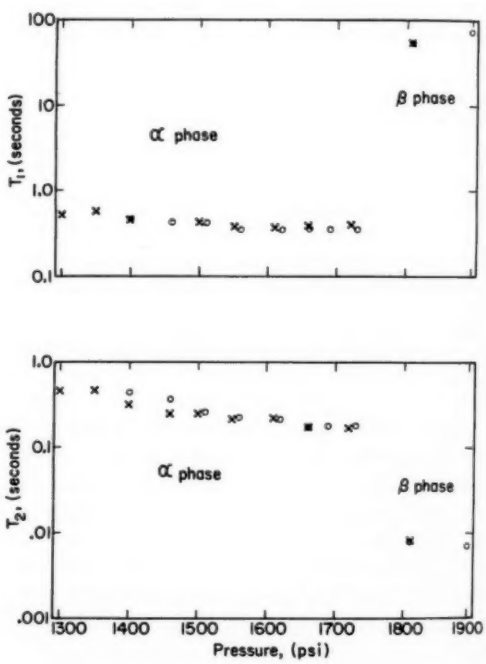

FIG. 1. Nuclear spin relaxation times in solid He^3 as a function of pressure at 2.2°K, indicating a discontinuity at the α-β phase boundary, discovered by Grilly and Mills (reference 4). Circles and crosses represent, respectively, data taken while increasing and decreasing the pressure.

2.2°K. The circles represent the data taken while increasing the pressure in steps and the crosses those taken while decreasing the pressure. At 1.37°K, however, the relaxation times displayed an exponential dependence on the pressure in the α phase and no discontinuity at the phase boundary. This may be seen in Fig. 2, where it may also be seen that T_1 passes through a minimum

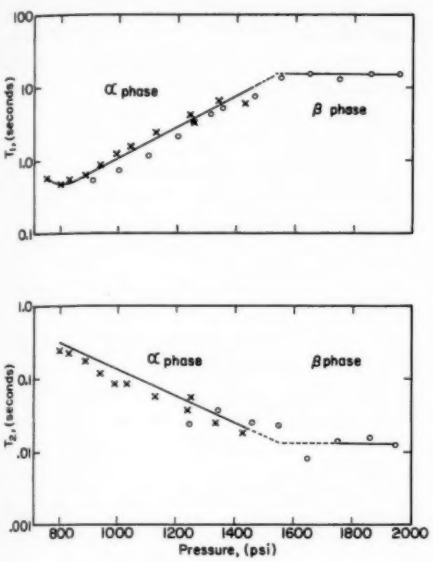

FIG. 2. Nuclear spin relaxation times as a function of pressure at 1.37°K, indicating a discontinuity in their rate of change with pressure at the α-β phase boundary. Circles and crosses represent, respectively, data taken while increasing and decreasing the pressure.

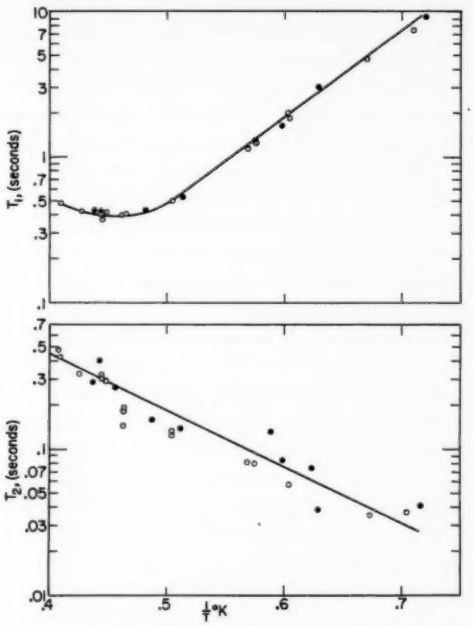

FIG. 3. Relaxation times as a function of the reciprocal of the temperature at a constant density of $\rho = 0.144$ g/cm^3, indicating an exponential dependence at temperatures here represented. Open circles and dots represent data taken on two different runs.

as the pressure changes near the liquid-solid phase boundary. Furthermore, there appears to be a discontinuity in the rate of change with pressure of T_1 and T_2. The pressure was measured to the accuracy obtainable with an Ashcroft Bourdon gauge.

Results consistent with this picture have been obtained at both higher and lower temperatures. Measurements of T_1 were made as low as 0.8°K, using a He^3 refrigerator, by both free-precession and steady-state methods. However, at this temperature T_1 was 26 minutes at 1300 psi, and apparently still increasing exponentially with pressure. Consequently, T_1 became so long as to make measurements at higher pressures difficult with the apparatus at hand. Since the discontinuity of the relaxation times vanishes near the temperature at which Grilly and Mills found the volume change approaching zero, a more detailed investigation of the temperature dependence of the effect is planned.

Several different constant-volume curves were measured. In general, in the temperature range investigated, the curves displayed the characteristics predicted by the semiclassical relaxation theories[5,6] (see Fig. 3). T_1 passes through a minimum as the temperature decreases and approaches an exponential dependence on $1/T$ near $T = 1.5$°K. T_2 displays an exponential dependence on $1/T$. This information is interpreted as an indication that in these temperature regions the relaxation is due primarily to an activated diffusion.[7] Thus $T_1 \propto 1/D$ and $T_2 \propto D$, where the diffusion coefficient D is of the form $D = D_0 \exp(-E/kT)$. Consequently, plotting the logarithms of T_1 and T_2 against the reciprocal of the temperature, the slope of the straight line part of the plot should be a measure of the activation energy E.

The energies determined from the T_1 data in this way were 12.3°, 14°, and 12.5°K for densities of 0.151, 0.144, and 0.142 g/cm^3. The T_2 values were consistently 3°K lower, for reasons which are not at present understood. The densities given above were estimated from the information given by Grilly and Mills and the known temperatures and pressures at which the constant-volume curves were started.

With a different apparatus some preliminary results have been obtained down to 0.15°K. A

single measurement of T_1 on the same constant-volume curve as that of Fig. 3 was about 40 minutes at 0.1°K. This is shorter by about 50 orders of magnitude than it would have been if it had continued to follow the exponential curve $T_1 = 3\times10^{-4}\exp(14/T)$ of Fig. 3. One complete set of T_1 measurements has been obtained between 0.7 and 0.14°K at a relatively low density (starting temperature and pressure 0.8°K and 500 psi). In this case T_1 was about 20% higher at 0.14°K and 0.7°K than at intermediate temperatures but seemed roughly constant at 0.55 ± 0.05 second between these limits.

In summary, the following information about solid He^3 is obtained from the results: (1) In the cases where there is a volume change associated with the α-β transition there are also large changes in the relaxation times. Since the relaxation in the α phase in these cases is primarily due to diffusion, there must also be changes in the diffusion coefficient across the phase boundary. (2) In the cases where the volume change approaches zero, there are no changes in the relaxation time across the boundary and, consequently, if there are changes in the diffusion coefficients, there must be compensating changes in other relaxation mechanisms. Even in these cases there remains a discontinuity in the rate of change with pressure of the relaxation times at the phase boundary. (3) For temperatures above 1.37°K the α phase behaves as an ordinary solid with a large diffusion coefficient. That is, the relaxation is determined by an activated diffusion of the atoms through the lattice. However, at temperatures somewhat below 1°K the relaxation is much too fast to be explained by the classical diffusive motion of the Bloembergen, Purcell, and Pound theory.

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[1]W. M. Fairbank and G. K. Walters, Symposium on Solid and Liquid Helium Three, 1957 (Ohio State University Press, Columbus, Ohio, 1958).

[2]W. M. Fairbank and G. K. Walters, Suppl. Nuovo cimento 9, 297 (1958).

[3]N. Bernardes and H. Primakoff, Phys. Rev. Letters 2, 290 (1959); 3, 144 (1959).

[4]E. R. Grilly and R. L. Mills, Ann. Phys. 8, 1 (1959).

[5]N. Bloembergen, E. Purcell, and R. V. Pound, Phys. Rev. 73, 679 (1948).

[6]H. C. Torrey, Phys. Rev. 92, 962 (1953); 96, 690 (1954).

[7]R. L. Garwin and H. A. Reich, Phys. Rev. 115, 1478 (1959). They have reported that the diffusion coefficient D in solid He^3 is 10^{-3} that of the liquid at the melting pressure. This is in rough agreement with the value of D calculated using the T_1 data reported here and Torrey's theory (reference 6).

SPECIFIC HEAT OF INDIUM BELOW 1°K*

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(Received March 16, 1960)

Heat capacity measurements on two indium specimens indicate that the T^3 term in the specific heat is smaller in the superconducting than in the normal state, contrary to the usual assumption that it is the same in both states. The expected temperature dependence in the normal state is

$$C_n = C_q + \gamma T + \alpha_n T^3 + \beta_n T^5, \qquad (1)$$

and in the superconducting state,

$$C_s = C_q + a\gamma T_c \exp(-bT_c/T) + \alpha_s T^3 + \beta_s T^5, \quad T < 0.7T_c. \qquad (2)$$

In each case, the first term, C_q, is the contribution from the nuclei due to an electric quadrupole interaction similar to that found in rhenium.[1] From the microwave resonance measurements of Hewitt and Knight[2] it follows that $C_q = 9.0\times10^{-4}$ millijoule/mole deg, which is 11% of C_s at 0.35°K, the lowest temperature reached in the measurement, and decreases to less than 1% of C_s at 0.6°K. The second term in C_s is the electronic specific heat, C_{es}, of a superconductor at temperatures below $0.7T_c$, and is less than 1% of C_s below half a degree. In this term, a and b are quasi-universal constants, and T_c is the transition temperature of indium (3.4°K). A

well-known expansion of the general expression for the electronic specific heat in the normal state yields γT for the first term, which is generally a very good approximation.

For estimating α_n, α_S, and γ, the specific heat data, now denoted by C_n and C_S, are plotted as $(C_n - C_q)/T$ and $(C_S - C_q)/T$ versus T^2 in Fig. 1. It is evident that the upper points can be represented by Eq. (1), and since they lie on a nearly straight line, $\beta_n T^5$ is very small in this region. A straight line through these points has a slope $\alpha_n = 1.55$ millijoule/mole deg^4 and yields $\gamma = 1.61$ millijoule/mole deg^2. Although our data agree with those of Clement and Quinnell[3] within the experimental error up to 4°K, they find $\alpha_n = 1.50$ and $\gamma = 1.81$, using the 1948 temperature scale and extrapolating their data below 1.7°K.

Under the usual assumption that the lattice specific heat in both states is the same, one would expect the lower set of points to have the limiting slope α_n as $T \to 0$. A line of slope α_n has been drawn through the origin, showing that $C_S < \alpha_n T^3$ for $T^2 < 0.6$. In fact, the lattice term in $(C_S - C_q)$

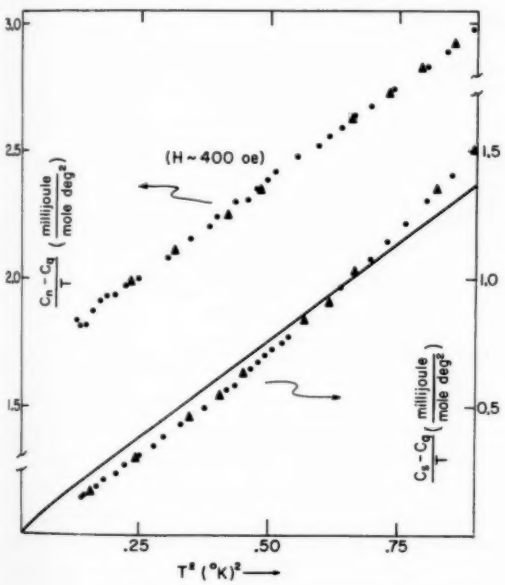

FIG. 1. Specific heat of two indium specimens in the normal and superconducting states after subtracting a nuclear contribution, C_q. The upper points, measured in a magnetic field of about 400 oersteds, are for the normal state. The solid line which is parallel to these points represents the lattice term in the normal state, and has been superimposed on the points measured in zero field, when the indium is superconducting.

cannot be greater than $1.1T^3$ at the lowest temperature reached (or not greater than $1.2T^3$ if the estimate of C_q were reduced to zero).

When these results were first obtained, the unexpectedly low value of γ and C_S led to a reinvestigation of the conceivable sources of systematic error.[4] Besides errors in the readily checked factors, such as heat capacity of the addenda (at most, 2% of the total), heater current and resistance, and duration of heating pulse, there can be hidden errors in the thermometry and heat input for which limits must be set. The temperature scales used to calibrate the carbon resistance thermometer are based on the 1958 vapor pressure scale for He^4 above 1°K and on the He^3 scale of Sydoriak and Roberts, adjusted to the 1958 He^4 scale. An iron ammonium alum salt thermometer provided an additional check on the calibration over most of the range, and replaced He^3 vapor pressure as the standard below 0.45°K. The largest possible error in C due to calibration, including effects of contamination of the He^3 by about 1% He^4, is estimated at 2%. The measurements below 1°K were done after midnight to avoid most of the sporadic heating from electrical and mechanical disturbances. The possibility was investigated that there may have been stray heat input during the heating periods. An extrapolation procedure corrected for the steady heat leak to the sample, which was not more than 3% of the input rate during a heating period. It is felt that the total systematic error in heat input is less than 1%. Afterward, a second sample was vacuum cast and annealed from 99.999% indium, and it produced the same results.

Below $0.7T_c$, C_S can be represented rather well by the empirical formula

$$C_q + a\gamma T_c \exp(-bT_c/T) + \alpha_S(T)T^3,$$

with $\alpha_S(T) \simeq 1.77e^{-1.9/T}$ when $a = 11$ and $b = 1.6$, which are estimates based on their values for other superconductors. Thus, $\alpha_S(T)$ diminishes toward lower temperatures, as if the lattice were becoming stiffer as the electron excitation decreases. Alternatively, C_S may be written with a negative term attributed to the influence of the electrons on the phonon spectrum. In either representation, the separation of C_S into pure "electronic" and "lattice" terms is convenient but imprecise nomenclature.

It has been pointed out by Chester[5] that as a consequence of the similarity rule for the critical

magnetic field [i.e., the critical field has the functional dependence $H_c = M^{-\alpha}F(M^{\alpha}T)$ on the isotopic mass M, where α is a constant equal to about 1/2], the difference in the specific heats $(C_n - C_s)$ must have the form $M^{-\alpha}f(M^{\alpha}T)$. The electronic terms have this dependence, but $\alpha_n \propto M^{3/2}$, so that the lattice term in the normal state does not have the required form, from which Chester concludes that $\alpha_n T^3$ must be cancelled by an identical term in C_s.

Though the similarity rule for isotopic mass holds accurately for some elements,[6,7] we are not aware of critical field measurements on the two natural isotopes of indium. However, Muench reported that another similarity rule, namely that the shape of the critical field curve is independent of pressure, does not hold for indium.[7] It would be desirable to have critical field measurements on the In isotopes, as well as extended heat capacity measurements on other soft superconductors having high T_c and relatively large lattice terms.

A detailed discussion of our data up to 4°K will appear in a forthcoming article.

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†National Science Foundation Cooperative Graduate Fellow.

[1]P. H. Keesom and C. A. Bryant, Phys. Rev. Letters 2, 260 (1959).

[2]R. R. Hewitt and W. D. Knight, Phys. Rev. Letters 3, 18 (1959).

[3]J. R. Clement and E. H. Quinnell, Phys. Rev. 92, 258 (1953).

[4]A discussion of previous measurements with the same apparatus is included in a paper by G. Seidel and P. H. Keesom, Phys. Rev. 112, 1083 (1958).

[5]G. V. Chester, Phys. Rev. 104, 883 (1956). See also P. M. Marcus and E. Maxwell, Phys. Rev. 91, 1035 (1953).

[6]C. A. Reynolds, B. Serin, and L. B. Nesbitt, Phys. Rev. 84, 691 (1951).

[7]Nils L. Muench, Phys. Rev. 99, 1814 (1955).

GIANT SPIN DENSITY WAVES

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The observations made in this paper call into serious question many details of the modern electron theory of metals. It will be shown below that, almost certainly, the Hartree-Fock ground state of a Fermi gas with Coulomb interactions is not the familiar Fermi sphere of occupied momentum states, but rather a state in which there are large static spin density waves, and in which large energy gaps exist in the single-particle excitation spectrum.

In order to emphasize the essential physical simplicity of the new low-energy states, we shall treat first a one-dimensional model. (Only translational freedom will be restricted to one dimension; the ordinary spin degrees of freedom will be retained.) The kinetic energy operator will be the usual one; and we shall assume that the repulsive interactions are delta functions:

$$V_{ij} = \gamma\delta(z_i - z_j). \tag{1}$$

The normal state of such a gas—N electrons in a box of length L—has all (plane wave) states occupied for $|k| \leq k_0 = \pi N/2L$. The total kinetic energy is $\frac{1}{2}NE_F$, where $E_F = \hbar^2 k_0^{\,2}/2m$; and the expectation value of the interactions (direct plus exchange) is $\gamma N^2/4L$. It is of interest to compare the energy of the normal state with that of the ferromagnetic state (all spins parallel):

$$W_{\text{ferro}} - W_{\text{normal}} = NE_F(1 - n^{-1}), \tag{2}$$

where

$$n = (N/L)/(2m\gamma/\pi^2\hbar^2), \tag{3}$$

a dimensionless quantity proportional to the electron density, N/L. The critical density, at which a transition between the normal and ferromagnetic states would occur, corresponds to $n = 1$. We shall prove, however, that the normal state is never the Hartree-Fock ground state, and that the ferromagnetic state is stable only for $n \leq \frac{3}{4}$.

We shall "begin" by writing down the self-consistent Hartree-Fock potential for the solutions of interest.

$$U(z) = 2gE_F(\sigma_x \cos qz + \sigma_y \sin qz), \tag{4}$$

where g and q are parameters to be determined later, and σ_x and σ_y are the usual Pauli matrices. The z direction need not be perpendicular to the (spin) x-y plane, but the spiral exchange potential (4) is perhaps easiest to visualize for that special case. This potential has the remarkable property that its only nonzero matrix elements are between the states (k, α) and $(k+q, \beta)$, where α and β are the spin-up and spin-down spin functions. The exact single-particle solutions for this potential are

$$\phi_k^{+} = [ge^{ikz}\alpha + (E_k^{+} - \omega_k)e^{i(k+q)z}\beta]/L^{1/2}[g^2 + (E_k^{+} - \omega_k)^2]^{1/2}, \tag{5a}$$

for spin-up states, and

$$\phi_k^{-} = [ge^{ikz}\beta + (E_k^{-} - \omega_k)e^{i(k-q)z}\alpha]/L^{1/2}[g^2 + (E_k^{-} - \omega_k)^2]^{1/2}, \tag{5b}$$

for spin-down states. The free-particle energies are $2E_F\omega_k$, where

$$\omega_k = \tfrac{1}{2}(k/k_0)^2, \tag{6}$$

and the perturbed single-particle energies, $2E_F E_k$, are given by

$$E_k^{+} = \tfrac{1}{2}(\omega_k + \omega_{k+q}) \pm [\tfrac{1}{4}(\omega_k - \omega_{k+q})^2 + g^2]^{1/2}, \tag{7}$$

together with a similar expression for E_k^{-} having q replaced by $-q$. The single-particle energy spectrum is shown in Fig. 1. Note the unusual feature that the energy gap, $4gE_F$, occurs on only one side of $k=0$ for a given spin. Near the energy gaps the real spin directions spiral in a plane perpendicular to the axis of spin quantization.

The N-electron wave function will be a single Slater determinant of wave functions (5). For any given q, the lowest energy is achieved if the occupied states satisfy $-\frac{1}{2}q \leq k \leq 2k_0 - \frac{1}{2}q$ for spin up, and $-2k_0 + \frac{1}{2}q \leq k < \frac{1}{2}q$ for spin down. The amplitude g must then be determined so that the Hartree-Fock equations are satisfied. Such a procedure yields the required value of g immediately. An alternative way is to calculate the expectation value of the total energy, and subsequently to minimize that energy with respect to g. A somewhat long, but straightforward, calculation gives the following result for the kinetic energy of the N-electron wave function, relative to that for the normal state:

$$\Delta T = NE_F(1-b)^2 + NE_F[2b - (4b^2+g^2)^{1/2} + (g^2/2b)\ln S], \tag{8}$$

where

$$S = [2b + (4b^2+g^2)^{1/2}]/g, \tag{9}$$

and $b = q/2k_0$. Since the total electron density of the new state is spatially uniform (as it is for the normal state), the only new contribution to the potential energy is an (algebraic) decrease of the total exchange energy, J. After another lengthy calculation, one finds

$$\Delta J = -(NE_F/4n)[(g/b)\ln S]^2. \tag{10}$$

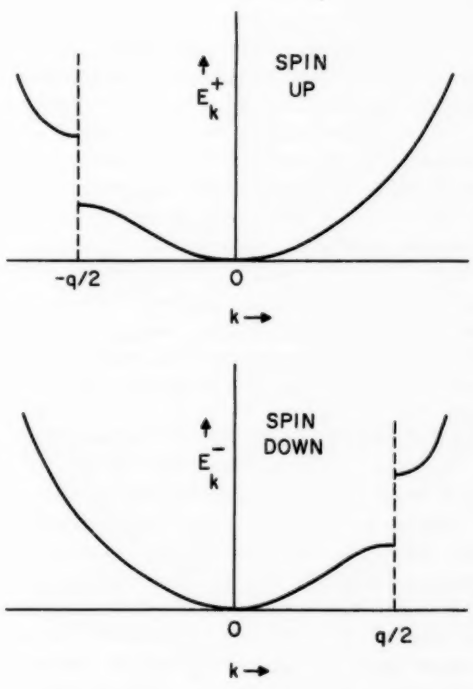

FIG. 1. Single-particle energy level spectrum for an electron gas with a giant spiral spin density wave.

An appreciable number of additional algebraic steps allows one to conclude that the sum of (8) and (10) is a minimum when

$$g = 2b/\sinh(2nb). \tag{11}$$

If this value is inserted into (8) and (10), the total energy of the new N-electron wave function relative to the normal state becomes

$$W = NE_F[1 + b^2 - 2b\coth(2nb)]. \tag{12}$$

This result is negative definite for $b = 1$, the value which corresponds to the same k-state occupation as the normal state. Consequently, the normal state is always a highly excited state. In fact, within the field of variation employed here, for $b = 1$, the normal state is an energy maximum.

The value (11) may be inserted into the wave functions (5), and the exchange potential operator computed according to its basic definition. One obtains a constant plus (4), the coefficient of the latter being in agreement with (11). Therefore, an N-electron state employing the functions (5) provides an exact solution of the Hartree-Fock equations for arbitrary b. One can show easily that the value of b which, in turn, minimizes (12) is less than unity. In fact, as n decreases (decreasing density or increasing interaction strength), $b \to 0$, at which value (12) and (2) become equal. This occurs at $n = \frac{3}{4}$. It should not be necessary to emphasize that the new lowest energy state has not been proved to be the Hartree-Fock ground state. However, we shall refer to it as the lowest (known) state. This lowest state has a spiral antiferromagnetic structure. The wavelength of the spiral is small, $\sim \pi/k_0$, for high density and gradually becomes larger, approaching ∞, as the density is reduced. Consequently the transition from spiral antiferromagnetism to ferromagnetism is a gradual one.

States above the energy gap can be occupied (e.g., by thermal excitations), but the spin directions of these states are out of phase with the spin density wave. As a result, the self-consistent amplitude of the spin density wave will be reduced, together with its contribution to the total energy. At a sufficiently high temperature—some fraction of the Fermi temperature—a second order phase transition will occur, and the normal state, with excitations, will (at last) become the state of minimum free energy. One can also construct nonspiral spin density wave states by employing an exchange potential $\propto \sigma_z \cos 2k_0 z$, instead of (4). These states, too, are always much lower than the normal state, but by an amount less than half that of the lowest spiral spin density wave.[1]

The physical reason why the spin density wave states are always lower than the normal state is, of course, the increase in magnitude of the exchange energy resulting from the local augmented parallelism of spins. The opposing term is of course the increase in kinetic energy. But the states most highly perturbed—those near the energy gap—are mixed with states of almost equal kinetic energy. Consequently this increase is sufficiently small to allow the exchange energy to dominate. In fact, in the limit of small g (considered again as a variational parameter), the ratio of exchange energy increase to kinetic energy increase approaches infinity (although only logarithmically). This observation makes it easy to prove that the normal state of a Fermi gas is unstable with respect to spin density wave formation for rather general repulsive interactions. Indeed, if the Fourier transform $V(k)$ of the interaction is large for small k, as it is for Coulomb interactions, the instability is enhanced, since the highly deformed single-particle states are close together in k space. (One must observe here that the fractional transfer of wave function amplitude to states across the Fermi sea is compensated by transfers in the reverse direction.)

The theory for a three-dimensional electron gas is almost identical to that given above for one dimension. It is necessary, however, to divide the electrons into at least three groups, and to allow each group to be deformed by its own spin density wave. For this reason the problem remains essentially one-dimensional. The occupied $\vec{k}$ states of each group will lie within a pyramidal cone (whose central axis is parallel to the wave vector of its spin density wave). For a given spin direction of a given group, the Fermi surface will be a plane on the energy gap side, and (approximately) a spherical polygon on the opposite side, as illustrated by the shaded region in Fig. 2. For strong interactions the complete Fermi surface will consist of a cube, at the surface of which is a large energy gap, and a sphere, at which $E(\vec{k})$ is continuous. Additional kinetic energy is required, of course, to occupy the unperturbed states in this way, but this will be more than compensated by the lower energy resulting from the spin density wave

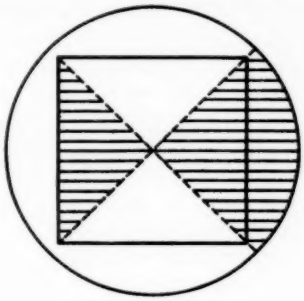

FIG. 2. Fermi surface for an electron gas with large repulsive interactions. The shaded region—a pyramidal cone—indicates the occupied states of a given spin for one (of three) groups of electrons. A large energy gap exists at the surface of the cube.

deformations. For weak interactions the inner Fermi surface will be a many-faced polyhedron, each pair of opposite faces arising from a spin density wave, and its volume will almost equal that of the sphere.

The foregoing description provides a means for "putting together" a three-dimensional electron gas in which the main effects of giant spin density waves are incorporated. The description is accurate only if the (off-diagonal) oscillatory part of the exchange potential between electrons of one group and those of another is neglected. Such effects, if included, will cause additional perturbation of the single-particle wave functions, will lower the total energy even further, and will prevent (fortunately) the boundary planes between different groups from having any sharp physical significance. Also, additional energy gaps in the spectrum of each group will be introduced by these perturbations. Consequently, the spherical part of the Fermi surface will lose some of its continuity. Such refinements need not be elaborated here, however.

A quantitative estimate of the energy gap at the surface of the cube for a typical metal with Coulomb interactions yields ~10 ev. (This estimate involves the formulation and solution of a complex nonlinear integral equation, and will be discussed elsewhere.) There seems to be little doubt that giant spin density waves should have numerous and profound consequences with regard to the properties of metals. Fortunately there remains some semblance of a Fermi sphere, but its states are no longer doubly degenerate. On the other hand, one can think of many experiments which might have revealed the existence of giant spin density waves, but have not: neutrons should suffer coherent magnetic diffraction in any metal, Langevin paramagnetism should not occur in metals or alloys, nuclear resonance lines should not be observed at their expected frequencies, if at all, etc. A possible escape from such enigmas is provided by the existence of collective excited states, of at least three varieties, relative to the spin density waves. Besides providing additional transport mechanisms, the excitation of collective modes (spin waves on the giant spin wave) will cause rapid fluctuations in the local spin density directions. It is by no means obvious that such fluctuations will be sufficiently rapid to avoid the unfortunate consequences just mentioned. Correlation energy differences are probably far too small to invert the new lowest state and the normal state in approximations beyond the Hartree-Fock scheme. Further research is necessary before one can decide whether or not a real paradox exists.

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[1]J. des Cloizeaux, J. phys. radium 20, 606 (1959) and 20, 751 (1959), has employed oscillating (nonspiral) spin density waves to describe the electronic structure of antiferromagnetic transition-metal oxides, an approach originally suggested by Slater [J. C. Slater, Phys. Rev. 82, 538 (1951)]. Here, the wave vector $\vec{q}$ is required to be half of a reciprocal lattice vector, and the state is energetically stable only if the Coulomb interactions are sufficiently large.

NUCLEAR GIANT DIPOLE RESONANCE*

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The collective oscillation of neutrons and protons in nuclei which can be excited by an electromagnetic field leads to a resonant response of nuclei[1-3] which is called the giant dipole resonance. Several classical descriptions of this motion have been given and more recently attempts have been made to describe the resonance starting from an independent-particle description of the nucleus.[4,5] It is the purpose of this note to clarify the relationship between those approaches. We shall restrict ourselves to a discussion of the classical treatment and shall give the detailed quantum mechanical treatment in a separate paper.

The giant dipole resonance is characteristic of finite nuclei, but the same type of relative displacement of neutrons and protons can occur in nuclear matter.[6,7] The description of the collective oscillation is very much simplified in the absence of boundaries, so we shall restrict ourselves to the case of nuclear matter. In the interest of simplicity, we consider an unperturbed state with equal neutron and proton densities. In this case the type of collective motion of interest is one in which the sum of neutron and proton densities remains constant but the neutron and proton densities separately fluctuate 180° out of phase. The shift in neutron and proton densities is opposed by the change in zero-point kinetic energy and in potential energy which tends to restore the neutron and proton density to equality. This effect, together with the inertial reaction of the neutrons and protons, leads in the small-amplitude region to harmonic oscillation of the densities.

Since the oscillation for large wavelengths involves a large number of nucleons, a classical description is appropriate. Any description starting from the single-particle approximation must also lead to the classical result for long-wavelength oscillations.[8]

To carry out the classical calculations, we first determine the single-particle energies, allowing for departures from the unperturbed neutron-proton densities. The single-particle energies are in general given in the K-matrix approximation[9] by

$$E_i = \frac{p_i^2}{2m} + \sum_j n_j (K_{ij,ij} - K_{ij,ji}), \tag{1}$$

with n_j the occupation number of the states of the medium. We rewrite the interaction term as

$$V_i^{(0)} + V_\tau, \tag{2}$$

with

$$V_i^{(0)} = \sum_j n_j^{(0)} (K_{ij,ij} - K_{ij,ji}), \tag{3}$$

$$V_\tau = \sum_j [n_j - n_j^{(0)}] (K_{ij,ij} - K_{ij,ji}), \tag{4}$$

and $n_j^{(0)}$ as occupation number in the unperturbed medium. Near the Fermi surface, the unperturbed potential energy $V_i^{(0)}$ can be written

$$V_i^{(0)} = \text{constant} + \tfrac{1}{2} p_i^2 \left(\frac{1}{m^*} - \frac{1}{m} \right), \tag{5}$$

with m^* the effective mass at the Fermi surface.

The remaining term in the potential energy as given in Eq. (4) is most simply evaluated if the sum over j is separated into neutron and proton sums and the K matrix is resolved into its spin and isotopic spin substates. In this evaluation we set the momentum p_i equal to p_F, since for long wavelengths of the collective oscillation the excitations lie close to the Fermi surface. The result is

$$(V_\tau)_{\text{neutrons}} = \frac{N-Z}{N+Z} v_\tau,$$

$$(V_\tau)_{\text{protons}} = \frac{Z-N}{N+Z} v_\tau, \tag{6}$$

with

$$v_\tau = \tfrac{1}{8}(N+Z) \Big[K_{\text{singlet, even}} + 3K_{\text{triplet, odd}} - K_{\text{singlet, odd}} - 3K_{\text{triplet, even}} \Big]_{\text{average}}. \tag{7}$$

The average indicated in Eq. (7) is

$$K_{\text{average}} = \int \frac{d\Omega_j}{4\pi} \left(K_{ij,ij} \right)_{p_i = p_j = p_F}. \tag{8}$$

A result of the form of Eq. (7) has previously been given by Brueckner and Gammel[9] in their evaluation of the nuclear symmetry energy.

Equations (1), (2), and (6) exhibit the important feature that the single-particle energies in general will depend on the fluctuation in neutron-proton densities typical of the collective density oscillations. This effect gives about half of the nuclear symmetry energy, and gives an appreciable shift in the frequency of the collective oscillation. This important contribution to the energy of the collective mode has not been included in previous discussions[4,5] starting from an independent-particle approximation.

Before we carry out the classical calculation,[10] we must determine the change of the zero-point kinetic energy as a function of the neutron and proton densities. For isotropic variation of the densities, the sum of the zero-point kinetic energy for neutrons and protons is

$$T=\sum_{i,N}\frac{p_i^2}{2m^*}+\sum_{i,Z}\frac{p_i^2}{2m^*}$$

$$=\frac{1}{n_D}\frac{p_F^2}{2m^*}\frac{(N-Z)^2}{N+Z}+\frac{3}{5}\frac{p_F^2}{2m^*}(N+Z), \quad (9)$$

with $n_D=3$ for the three dimensions excited in the medium. The mean change per nucleon then is[3] (for $N+Z$ constant)

$$\left(\frac{p_i^2}{2m^*}\right)_{\text{average}}=\frac{\partial T}{\partial N}=\frac{1}{n_D}\frac{p_F^2}{m^*}\frac{N-Z}{N+Z}, \quad \text{neutrons}$$

$$=\frac{\partial T}{\partial Z}=\frac{1}{n_D}\frac{p_F^2}{m^*}\frac{Z-N}{N+Z}, \quad \text{protons.} \quad (10)$$

If the change in density is due to a one-dimensional motion, then only one degree of freedom is excited and n_D in Eqs. (9) and (10) is equal to one. The excitation of all three degrees of freedom takes place in ordinary sound propagation, for example, because of collisions. In a plasma oscillation, however, collisions are missing (except for weak damping effects) and the oscillation is one-dimensional only. The three-dimensional excitation can also occur because of cooperative pairing effects[11] analogous to those in superconductors; such effects are, however, expected to be absent in nuclear matter except for very long wavelengths. Thus we shall set $n_D=1$ in Eq. (10). In this we differ from the phenomenological treatment of Steinwedel and Jensen[3] who used the empirical nuclear symmetry energy which is obtained, of course, for isotropic density changes in nuclei.

To obtain the classical equations of motion, we now replace in Eqs. (6) and (10)

$$(N-Z)/(N+Z)=(\rho_N-\rho_Z)/(\rho_N+\rho_Z), \quad (11)$$

with ρ_N and ρ_Z the neutron and proton densities. This replacement is valid as long as the collective oscillation has wavelength large enough to include many nucleons. The classical equations then are

$$m_t\frac{d\vec{v}_Z}{dt}=-\vec{\nabla}\left[2u_\tau\left(\frac{\rho_Z-\rho_N}{\rho_N+\rho_Z}\right)\right],$$

$$m_t\frac{d\vec{v}_N}{dt}=-\vec{\nabla}\left[2u_\tau\left(\frac{\rho_N-\rho_Z}{\rho_N+\rho_Z}\right)\right], \quad (12)$$

with v_Z and v_N the average velocities of displacement of the protons and neutrons and

$$u_\tau=\frac{p_F^2}{2m^*}+\frac{1}{2}v_\tau. \quad (13)$$

If the factor of 1/3 in Eqs. (9) and (10) were included in the kinetic energy term in Eq. (12), v_τ would be the usual nuclear symmetry energy K. Since the calculation of Brueckner and Gammel gives $v_\tau\cong\frac{1}{3}p_F^2/m^*$, we have the approximate relation

$$u_\tau\cong 2K. \quad (14)$$

The effect of this change on the oscillation frequency is apparent in Eq. (20), which shows that the frequency varies at $(u_\tau)^{1/2}$.

In Eq. (12) we mean by m_t the mass appropriate to the collective displacement of the neutrons relative to the protons. This will not in general be equal to m^* since the excitation is collective, the neutrons for example not changing their motion relative to the other neutrons as they do when singly excited. However, since most of the potential energy of a neutron is due to its interaction with the protons, it is probable that m_t is nearly equal to m^* and we shall take it equal in the following.

We now linearize Eq. (12) for small fluctuations, writing

$$\vec{v}_i=\partial\vec{\xi}_i/\partial t,$$

$$\rho_i=\tfrac{1}{2}\rho_0+\rho_i', \quad (15)$$

and also making use of the fact that $\rho_N + \rho_Z = \rho_0$ is constant. The result is

$$\tfrac{1}{2}\rho_0 \operatorname{div}(\partial\vec{\xi}_i/\partial t) + (\partial\rho_i'/\partial t) = 0 \qquad (16)$$

or

$$\rho_i' = -\tfrac{1}{2}\rho_0 \operatorname{div}\vec{\xi}_i. \qquad (17)$$

Equation (12) then becomes (we drop terms quadratic in the velocities, and set $m_t = m^*$)

$$m^* \partial^2\vec{\xi}_Z/\partial t^2 = u_\tau \vec{\nabla} \operatorname{div}(\vec{\xi}_Z - \vec{\xi}_N),$$

$$m^* \partial^2\vec{\xi}_N/\partial t^2 = u_\tau \vec{\nabla} \operatorname{div}(\vec{\xi}_N - \vec{\xi}_Z). \qquad (18)$$

Now assuming one-dimensional motion and combining the neutron and proton equations, we find

$$\frac{\partial^2(\xi_Z - \xi_N)}{\partial t^2} = \frac{2u_\tau}{m^*}\frac{\partial^2(\xi_Z - \xi_N)}{\partial x^2}. \qquad (19)$$

This is a wave equation of usual form, giving the dispersion relation between frequency and momentum q,

$$\omega^2 = \frac{2u_\tau}{m^*}\left(\frac{q}{\hbar}\right)^2, \qquad (20)$$

or, from Eq. (13),

$$\hbar\omega = \frac{qp_F}{m^*}\left(1 + \frac{m^* v_\tau}{p_F^2}\right)^{1/2}. \qquad (21)$$

Since $v_t \cong \frac{1}{3}p_F^2/m^*$, the correction to $\hbar\omega$ is about 15%. It is not possible to extrapolate this result to finite nuclei, but it is clear that this collective effect can lead to appreciable elevation of the eigenvalue above the single-particle value.

The result previously obtained by Landau[6] and by Glassgold, Heckrotte, and Watson[7] is of the form

$$\hbar\omega = (qp_F/m^*)(1 + e^{-\Delta}), \qquad (22)$$

with

$$\Delta = 4\pi^2/[V(0)m^*p_F], \qquad (23)$$

and $V(0)$ the matrix element of the two-particle interaction for small momentum transfer. The correction term $e^{-\Delta}$ arises from the collective character of the motion, the derivation of this term following the usual treatment of the collective excitations in the electron gas. This treatment omits, however, the collective effect we have considered. The estimate by Glassgold et al., using reasonable nuclear matrix elements, gives a value for $e^{-\Delta}$ of about 1% so that a negligible shift in the frequency occurs from this effect. This is in contradiction with the suggestion by Brown and Bosterli[5] that the large shift in the giant dipole frequency from the single-particle value can be attributed to this term.

In summary, we wish to re-emphasize the large role played in the nuclear symmetry energy by the change in potential energy resulting from density changes. This well-known effect necessarily appears in a classical discussion of the giant dipole resonance. Finally, as we have pointed out, a correct discussion of the collective motion starting from a single-particle approximation will lead to the classical result.

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*Supported in part by a grant from the U. S. Atomic Energy Commission.

[1]M. Goldhaber and E. Teller, Phys. Rev. 74, 1046 (1948).

[2]J. H. D. Jensen and H. Steinwedel, Z. Naturforsch. 59, 343 (1950) and Z. Naturforsch. 59, 413 (1950).

[3]H. Steinwedel and J. H. D. Jensen, Phys. Rev. 79, 1019 (1950).

[4]D. H. Wilkinson, Physica 22, 1039 (1956).

[5]G. E. Brown and M. Bosterli, Phys. Rev. Letters 3, 472 (1960).

[6]L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 59 (1957) [translation: Soviet Phys. -JETP 5, 101 (1957)], has also given a description of other types of collective motion in the uniform medium.

[7]A. E. Glassgold, W. Heckrotte, and K. M. Watson, Ann. Phys. 6, 1 (1959).

[8]This situation also arises in the electron gas. The relation of the classical result to the single-particle description has been discussed in detail by D. Bohm and D. Pines, Phys. Rev. 92, 609 (1953) and K. Sawada, K. A. Brueckner, N. Fukuda, and R. Brout, Phys. Rev. 108, 18 (1957).

[9]See, for example, K. A. Brueckner and J. L. Gammel, Phys. Rev. 109, 1023 (1958). This paper also gives references to the related work in this field.

[10]The classical treatment has been given in detail in reference 2, and we only sketch the derivation.

[11]This point is discussed in detail by N. Bogoliubov, V. Tolmacher, and D. Shirkov, A New Method in the Theory of Superconductivity (Academy of Sciences of U.S.S.R., Moscow, 1958; Consultant's Bureau, Inc., New York, 1959). N. Bogoliubov, Uspekhi Fiz. Nauk 67, 549 (1959) [translation: Soviet Phys. – Uspekhi 67(2), 236 (1959)]. P. W. Anderson, Phys. Rev. 112, 1900 (1958). T. Soda (unpublished paper, 1959).

ORDER OF LEVELS IN THE SHELL MODEL AND SPIN OF Be^{11}*

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It is often assumed that the order of single-nucleon levels is the same for neighboring odd nuclei. This, however, is not always the case if the residual (effective) two-body interactions are taken into account. The detailed consideration of these interactions leads sometimes to interesting "competition" between orbits. For example, the order of filling of neutron shells may depend on the proton configuration. In this note, we discuss from this point of view the ground-state spin of Be^{11}. The experimental data recently obtained[1] indicate a spin and parity 1/2+. This assignment seems to be very unexpected since the standard order of levels in the shell model suggests a 1/2- assignment. We shall show that a 1/2+ spin of Be^{11} due to a last $2s_{1/2}$ neutron is plausible and even preferred on the basis of the detailed quantitative scheme of the shell model.

In C^{13} the $s_{1/2}$ level is 3.09 Mev above the $p_{1/2}$ ground state. However, there are nuclei in which the $s_{1/2}$ orbit is below the $p_{1/2}$ orbit (e.g., Be^{9} and also Be^{10}). It is, therefore, difficult to decide what is the order of these orbits in Be^{11} without a quantitative calculation. Such a calculation is demonstrated in Fig. 1. The first two levels in B^{12} are due to the coupling of one $p_{3/2}$ proton (hole) to the $p_{1/2}$ neutron. The next two levels are similarly obtained by coupling to an $s_{1/2}$ neutron. The spin and parity assignments are based on the B^{12} data as well as on the data for the first $T=1$ levels in C^{12}.[2] The center of mass of the two $p_{1/2}$ levels lies at 0.59 Mev while that of the two $s_{1/2}$ levels lies at 2.03 Mev above the ground state.

A linear extrapolation of the $p_{1/2}$ - $s_{1/2}$ difference in C^{13} (3.09 Mev) and the corresponding difference between centers of mass in B^{12} (1.44 Mev) gives the predicted difference in Be^{11}. The $s_{1/2}$ state is predicted to be the ground state, 0.21 Mev below the $p_{1/2}$ level.

The linear extrapolation in Fig. 1 is based on the fact that the interaction of a j' neutron with two j protons coupled to $J=0$ is given by

$$\langle j^2(J=0)j'|V_{1n}+V_{2n}|j^2(J=0)j'\rangle = 2\sum_{J=|j-j'|}^{J=j+j'}(2J+1)\langle jj'J|V|jj'J\rangle \Big/ \sum_{J=|j-j'|}^{J=j+j'}(2J+1). \quad (1)$$

Thus, the change in interaction energy of a $p_{1/2}$ or $s_{1/2}$ neutron when two $p_{3/2}$ protons are removed is twice the change due to the removal of one $p_{3/2}$ proton. This latter change is given by the position of the center of mass as expressed in (1). An analogous case which demonstrates the usefulness of the linear extrapolation is given in Fig. 2. The competition between the $d_{5/2}$ and $s_{1/2}$ orbits is clearly seen. The linear extrapolation from O^{17} and N^{16} gives for C^{15} an $s_{1/2}$ ground state with a $d_{5/2}$ level at 0.58 Mev above it. This agrees very well with the measured spin of C^{15} and the position of its first excited state (0.66 Mev).[2]

Our result about the C^{15} spin was actually based not only on O^{17} and N^{16} but on many more data.[3] In the present case there are not enough data and the result obtained above on Be^{11} is not that certain. The error involved in our estimate can be roughly obtained by considering the Be^{11} binding energy. The separation energy of the $s_{1/2}$ neu-

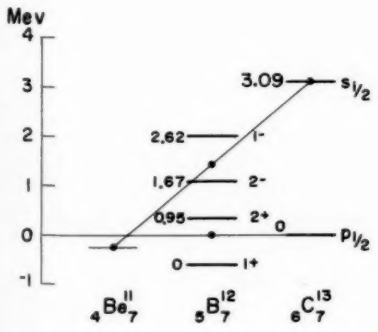

FIG. 1. Competition between $s_{1/2}$ and $p_{1/2}$ levels.

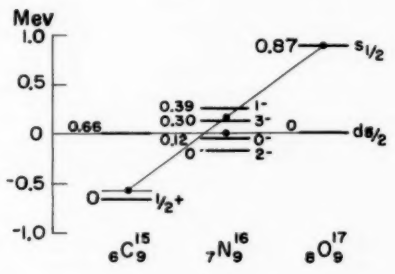

FIG. 2. Competition between $d_{5/2}$ and $s_{1/2}$ levels.

tron in C^{13} is 1.86 Mev. That of the center of mass of the 2- and 1- levels in B^{12} is 1.34 Mev. The linear extrapolation to Be^{11} yields 0.82 Mev for the $s_{1/2}$ neutron separation energy. This value (i.e., the binding energy difference between Be^{11} and Be^{10}) leads to an energy difference between Be^{11} and B^{11} of 11.20 Mev as compared to the experimental value[1] of 11.48 ± 0.15 Mev. The only definite prediction is thus that the 1/2+ and 1/2- levels should be very close in Be^{11}. It would not be surprising if the Be^{11} spin is 1/2-. However, the expectation that the spin and parity of the ground state of Be^{11} are indeed 1/2+, as suggested by the experiment, is at least as firm.

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[1]D. H. Wilkinson and D. E. Alburger, Phys. Rev. 113, 563 (1959).

[2]F. Ajzenberg-Selove and T. Lauritsen, Nuclear Phys. 11, 1 (1959).

[3]I. Unna and I. Talmi, Phys. Rev. 112, 452 (1958).

PANOFSKY RATIO*

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The Panofsky ratio, P, which is defined as the relative probability of mesonic to radiative capture of π^- mesons from the K shell in hydrogen,

$$P = \frac{\text{Rate } (\pi^- + p \to n + \pi^0)}{\text{Rate } (\pi^- + p \to n + \gamma)},$$

has been measured many times by various people as is shown in Table I. In those experiments, the two reactions were detected either by the use of a pair spectrometer magnet, a large glass Čerenkov counter, or a cloud chamber. The values obtained for P vary quite widely and are statistically incompatible. The Panofsky ratio can be connected through detailed balancing arguments to low-energy charge exchange scattering and photomeson production as has been pointed out by Marshak[1] and Fermi and Anderson.[2] In making the comparison it is necessary to know the cross section at threshold for charge exchange scattering, the cross section at threshold for π^+ photoproduction on hydrogen, and the ratio of π^-/π^+ photoproduction from a single nucleon which is related to photoproduction of π^-/π^+ in deuterium. Agreement or disagreement can be attained between these phenomena and P depending upon extrapolation pro-

Table I. List of previous measurements of the Panofsky ratio.

P	Reference
0.94 ± 0.30	W. K. H. Panofsky, R. L. Aamodt, and J. Hadley, Phys. Rev. 81, 565 (1951).
1.10 ± 0.50	C. P. Sargent, R. Cornelius, M. Rinehart, L. M. Lederman, and K. Rogers, Phys. Rev. 98, 1349 (1955).
1.50 ± 0.15	J. M. Cassels, G. Fidecaro, A. Wetherell, and J. R. Wormald, Proc. Phys. Soc. (London) A70, 405 (1957).
1.60 ± 0.17	J. Kuehner, A. W. Merrison, and S. Tornabene, Proc. Phys. Soc. (London) 73, 545 (1959).
1.87 ± 0.10	J. Fischer, R. March, and L. Marshall, Phys. Rev. 109, 533 (1958).
1.46 ± 0.10	L. Koller and A. M. Sachs, Phys. Rev. 116, 760 (1959).

cedures adopted for the threshold reactions and the manner in which the deuterium results are corrected to that of production from a single nucleon. The difficulties encountered in these procedures have been discussed by many authors.[3-6] The present experiment was undertaken in order to determine P to a greater accuracy and by a means free from the difficulties of the previous measurements.

It was first pointed out by Dalitz[7] that one of the γ rays from the π^0 decay can be internally converted with a probability ~1/160. Kroll and Wada[8] extended this idea to the $(n+\gamma)$ reaction and calculated, among other things, the probability of the internal conversion of this γ ray which was ~1/140. Therefore, by counting the number of internal pairs of each type [i.e., from $n+\pi^0$, and $(n+\gamma)$], the Panofsky ratio can be determined since

$$P=\frac{\rho_\gamma}{\rho_{\pi^0}}\frac{N_{\pi^0}}{N_\gamma},$$

where N_γ = number of internal pairs from $(n+\gamma)$, N_{π^0} = number of internal pairs from $(n+\pi^0)$, ρ_γ = internal conversion coefficient for γ for $(n+\gamma)$, and ρ_{π^0} = internal conversion coefficient for either γ for $(n+\pi^0)$.

π^- mesons from the Nevis Cyclotron were slowed down by a polyethylene absorber and stopped in a liquid hydrogen bubble chamber 12 in. in diameter and 6 in. in depth. The chamber was placed in a magnetic field of 8.8 kgauss for 1/3 of the pictures and 5.5 kgauss for the remaining 2/3 of the pictures. Approximately 200 000 pictures were taken for this analysis with ~5-10 stops per picture. A fiducial region which encompassed ~50% of the cross-sectional area of the chamber was chosen and only π^- mesons which stopped in this region were considered. All pictures were scanned for such stopped π^-'s which gave rise to internally converted pairs such as shown in Fig. 1. The vector momentum and energy, with their associated measurement and multiple-scattering errors, of each pair were determined as well as the depth position of the vertex in the chamber. 7000 such events were measured. A depth and dip criterion was further applied which consisted in using only those events whose vertex was at least 3 cm from top and bottom glass windows, and where the cosine of the dip angle of the vector momentum of the pair was less than 0.8. These restrictions insured the measurability of the events in an unbiased manner but also reduced their number to 4199.

A separation into the two categories, $(n+\pi^0)$ or $(n+\gamma)$, was made possible by the use of a momentum versus energy plot as shown in Fig. 2. The 1090 $(n+\gamma)$ events fell along the designated line; the 3065 $(n+\pi^0)$ events fell, within their

FIG. 1. Bubble chamber photograph of an internally converted electron positron pair.

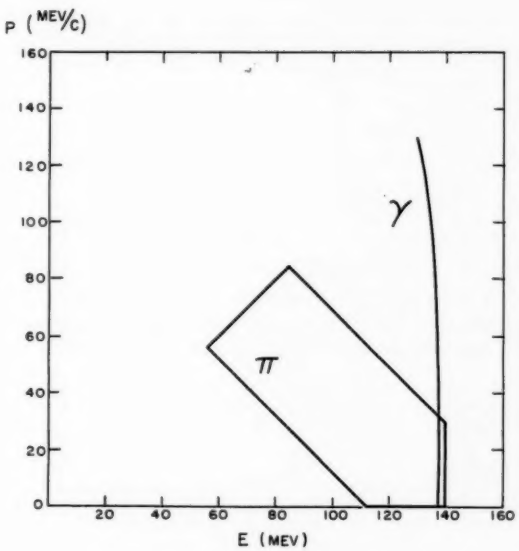

FIG. 2. Separation graph showing $n+\pi^0$ region and $n+\gamma$ line plotted as a function of the pair energy and momentum.

errors, in the indicated box; while 44 events were in the overlapping region or in between the two regions but with sufficiently large errors so that they overlapped both regions. The energy distribution of all the pairs is shown in Fig. 3. The histogram refers to the experimental points and the smooth curve to the theoretically expected distribution with an experimental Gaussian resolution of 7.0% folded in. The 44 other events were apportioned according to this curve, giving

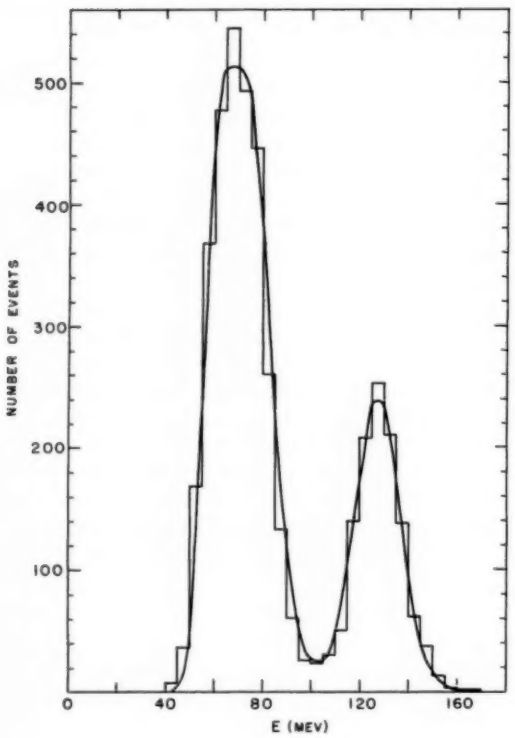

FIG. 3. Energy distribution of the internally converted pairs. The histogram refers to the experimental data and the smooth curve to the theoretical fit.

$N_{\pi^0} = 3071 \pm 55$, $N_\gamma = 1128 \pm 34$. Therefore

$$P = \frac{(3071 \pm 55)}{(1128 \pm 34)} \times 0.594 = 1.62 \pm 0.06.$$

The values for the conversion coefficients are those given by Joseph.[9] This calculation is similar to that of Kroll and Wada but electromagnetic corrections which constitute 1% correction to both coefficients have been included. The error quoted for the Panofsky ratio includes a 1% error on these coefficients as well as the statistical error on the number of events and in the apportionment of the 44 events. It should be emphasized that the above result depends upon the accuracy of the calculation of the conversion coefficients. Further details concerning the above analysis, as well as a comparison of these internal pairs with the theory as given by references 8 and 9, will soon be presented.

I wish to thank the Columbia Bubble Chamber Group composed of Dr. A. Prodell, Dr. R. Plano, Dr. M. Schwartz, and Dr. J. Steinberger for aid in taking the pictures; the scanners Irene Gardner, John Impeduglia, Dina Goursky, and Alex Rytow for their painstaking work; and the computer group composed of Dr. D. Tycko and D. Burd for programming the IBM 650.

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[1]R. E. Marshak, Phys. Rev. 82, 313 (1951).

[2]H. L. Anderson and E. Fermi, Phys. Rev. 86, 794 (1952).

[3]A. M. Baldin, Nuovo cimento 8, 569 (1958).

[4]A. M. Baldin and P. K. Kabir, Nuovo cimento 9, 547 (1958).

[5]M. Cini, R. Gatto, E. L. Goldwasser, and M. Ruderman, Nuovo cimento 10, 243 (1958).

[6]M. J. Moravcsik, Phys. Rev. 111, 1657 (1958).

[7]R. H. Dalitz, Proc. Phys. Soc. (London) A64, 667 (1951).

[8]N. Kroll and W. Wada, Phys. Rev. 98, 1355 (1955).

[9]D. W. Joseph, Nuovo cimento (to be published).

CONSTRUCTION OF COUPLED SCATTERING AND PRODUCTION AMPLITUDES SATISFYING ANALYTICITY AND UNITARITY*

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(Received April 1, 1960)

The methods of Omnes[1] and of Chew and Mandelstam[2] for constructing single-channel (e.g., partial wave) scattering amplitudes, production amplitudes, and vertex functions on the basis of analyticity and unitarity have been extended to the case of n^2 coupled amplitudes $T_{ij}(\omega)$ representing the production and scattering among n different channels, each of which may have a different threshold energy.

In the limit of infinitely short-range interactions, the analyticity properties of any partial-wave production or scattering amplitude $T_{ij}(\omega)$ as a function of the complex energy[3] ω may be expected to consist only of a single branch cut along the positive axis beginning at a threshold energy ω_i, corresponding to the lowest energy intermediate state. Effects of interactions of finite range in this picture manifest themselves in the appearance of additional branch cuts and poles in the "unphysical region" of negative—or possibly complex—values of ω. However, for finite-range interactions involving particles with a sufficiently compact structure,[4] the cuts for positive real ω which appear in T_{ij} will be surrounded by at least some limited region R of analyticity.

In the method of Chew and Mandelstam, one writes the scattering amplitude for one channel as

$$T(\omega) = \gamma(\omega)/D(\omega), \tag{1}$$

with $D(\omega)$ analytic everywhere except for the cut in the physical region, i.e., the positive real ω axis (and possibly poles for bound states). $\gamma(\omega)$ is analytic in the region R, including the positive real axis. In order to satisfy unitarity, one finds

$$D(\omega) = 1 - \frac{1}{\pi}\int_{\omega_0}^{\infty} \frac{d\omega'\,\rho(\omega')\gamma(\omega')}{\omega' - \omega}. \tag{2}$$

The physical content is expressed then by means of a smooth function $\gamma(\omega)$ and the task of a theory is to compute it. However, approximations on γ have the property that analyticity and unitarity are preserved. The Chew-Mandelstam method consists of restricting singularities of $\gamma(\omega)$ to those nearest the physical region of positive real ω and matching them to the presumably known discontinuities in T. Alternatively one computes $\gamma(\omega)$ by perturbation theory, such a procedure yielding the determinantal method of Baker.[5]

In the case of n coupled channels, the unitarity conditions for the amplitudes T_{ij} and, correspondingly, vertex functions F_j are:

$$\frac{1}{2i}\left[T_{ij}(\omega + i\epsilon) - T_{ij}(\omega - i\epsilon)\right] = \sum_k \rho_k(\omega) T_{ik}(\omega - i\epsilon) T_{kj}(\omega + i\epsilon), \tag{3a}$$

$$\frac{1}{2i}\left[F_j(\omega + i\epsilon) - F_j(\omega - i\epsilon)\right] = \sum_k \rho_k(\omega) F_k(\omega - i\epsilon) T_{kj}(\omega + i\epsilon). \tag{3b}$$

The $\rho_k(\omega)$ are phase space factors which vanish below the relevant production threshold. It has been assumed that the unitarity condition may be continued into any of the "unphysical regions" along the positive ω axis which are involved, which appears to be correct if no anomalous thresholds are present.[6] Our problem then is to express $T_{ij}(\omega)$ and $F_j(\omega)$ in terms of functions $\gamma_{ij}(\omega)$ which are analytic in some region R surrounding it. We define the matrix $d_{ij}(\omega)$ by the relation

$$d_{ij}(\omega) = \delta_{ij} - \frac{1}{\pi}\int_{\omega_i}^{\infty} \frac{d\omega'\,\rho_i(\omega')\gamma_{ij}(\omega')}{\omega' - \omega}. \tag{4}$$

d_{ij} is analytic except for the cut on the positive axis beginning at the threshold energy ω_i. A natural generalization of Eq. (1) is to consider the matrix expression

$$T_{ij} = \sum_k \frac{\gamma_{ik}(\omega) D_{(jk)}}{D}, \tag{5}$$

with

$$D = \det d_{ij} \tag{6}$$

and the minor $D_{(jk)}$ defined by

$$\delta_{ij} D = \sum_k d_{ik} D_{(jk)}. \tag{7}$$

Substituting Eq. (5) into the right-hand side of Eq. (3) and using Eq. (4) and Eq. (7), one verifies that the solution Eq. (5) satisfies the unitarity condition Eq. (3a) provided r_{ij} is analytic in R.

If the scattering solutions [i.e., $r_{ij}(\omega)$] are known, the solutions to the problem of coupled form factors Eq. (3b) are

$$F_j(\omega) = \sum_k \frac{f_k D(jk)}{D}, \qquad (8)$$

with the f_k analytic in R. If F_j has no other singularities, the f_k are constants.

A practical procedure of handling these equations is to expand r_{ij} and D in the coupling parameter. Alternatively one may approximate the singularities of r_{ij}—and thus the corresponding singularities of T_{ij}—by poles, and fitting the residues of r_{ij} to the singularities of T_{ij}. If the singularities of T_{ij} are known, the procedure reduces to a system of coupled algebraic equations.

Applications of these methods to double pion production, associated production, and K^--N scattering are in progress.

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*Supported in part by the U. S. Air Force through the Air Force Office of Scientific Research.

[1]R. Omnes, Nuovo cimento 8, 316 (1958).

[2]G. F. Chew and S. Mandelstam, University of California Radiation Laboratory Report UCRL-8728, April, 1959 (unpublished).

[3]We may equally well use these arguments for other variables such as ω^2 or the square of the center-of-mass momentum.

[4]By this we mean that cases involving "anomalous thresholds" are excluded. See, however, reference 6.

[5]M. Baker, Ann. Phys. 4, 271 (1958).

[6]S. Mandelstam, Phys. Rev. Letters 4, 84 (1960).

EFFECT OF PION-PION RESONANCES ON $\pi^- p$ INTERACTIONS*

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(Received April 8, 1960)

Some recent experiments[1,2] on the production of π mesons by π mesons at about a Bev have led to contradictory results with respect to the spectra of emitted pions: in one,[1] the π^- from the process $\pi^- + p \to \pi^- + p + \pi^0$ emerges predominantly fast whereas in the other,[2] the π^- is predominantly slow. The former result is quite adequately explained by the isobar model[3] (even if the possibility of excitation of the $T = 1/2$ isobar is included) whereas the latter results are directly contradictory to this model. It may be that the pion spectrum is extremely energy sensitive, and that the two sets of data differ because the incident pion energies are sufficiently different, 960 Mev and 1 Bev, respectively. It is useful, therefore, to find a model which could leave the interpretation of the first pion spectrum unaltered but which could fit the second spectrum owing to the rapid onset of another process.

We would like to indicate here a possible explanation based on a model which is very closely tied to the isobar model and which retains much of its conceptual simplicity: Namely, the inclusion of a π-π interaction predominantly in a $T = 1$ state[4] along with the production of an isobar is considered to be the mechanism for the production of pions.

The picture we use is that the incoming pion collides with a pion in the cloud, scattering, causing one of the pions to be emitted, and leaving the nucleon in an excited state which then decays to the ground state with the emission of the second pion. We will consider the possibility that both the 3/2 and 1/2 isobars can be excited. We can easily calculate the energy of the emerging pions in either case and using the mean of the energies from the two isobars as that of the peak in the spectrum, we have a momentum of 340 Mev/c for 950-Mev incident pions.

On the basis of the above picture, we can readily calculate the ratio of fast to slow π^-: we then obtain, for the case of π-π interaction in $T = 1$ state,

$$\frac{N(\pi^- \text{ fast})}{N(\pi^- \text{ slow})} = \frac{\frac{4}{3}(\frac{1}{6})^2 |a_{1/2}|^2}{\frac{1}{3}(\frac{1}{2})^2 |a_{1/2}|^2 + \frac{1}{3}(\frac{1}{6})^2 |a_{3/2}|^2}$$

$$= \tfrac{4}{9}\left(\frac{|a_{1/2}|^2}{|a_{1/2}|^2 + \frac{1}{9}|a_{3/2}|^2}\right),$$

$$\tfrac{4}{9} \geq N(\pi^- \text{ fast})/N(\pi^- \text{ slow}) \geq 0. \qquad (1)$$

Of course, there is some production by mechanisms other than that proposed here, and thus these limits may be changed somewhat.

It is of some interest to see the effect of other states in the π-π interaction on this ratio and whether, if the above mechanism is accepted, a $T=1$ "resonance" is needed. Thus, if the $T=2$ state is dominant,

$$\frac{N(\pi^- \text{ fast})}{N(\pi^- \text{ slow})} = \frac{\frac{4}{3}(\frac{1}{5})^2|a_{3/2}|^2 + \frac{4}{3}(\frac{5}{18})^2|a_{1/2}|^2}{\frac{1}{3}(\frac{1}{15})^2|a_{3/2}|^2 + \frac{1}{3}(\frac{5}{18})^2|a_{1/2}|^2},$$

$$16 \geq N(\pi^- \text{ fast})/N(\pi^- \text{ slow}) \geq 4, \qquad (2)$$

or if the $T=0$ is dominant,

$$\frac{N(\pi^- \text{ fast})}{N(\pi^- \text{ slow})} = \frac{\frac{4}{3}(\frac{1}{9})^2|a_{3/2}|^2 + \frac{4}{3}(\frac{1}{9})^2|a_{1/2}|^2}{\frac{4}{3}(\frac{1}{9})^2|a_{3/2}|^2 + \frac{4}{3}(\frac{1}{9})^2|a_{1/2}|^2} = 1. \qquad (3)$$

In neither case could one obtain a predominance of slow π^- mesons.

It is interesting to note that for the $T=1$ case, the π^+ meson is <u>always</u> fast by the above mechanism in the reaction $\pi^- + p \rightarrow n + \pi^+ + \pi^-$.

We can also consider the total number of $\pi^+ + \pi^-$ reactions to $\pi^- + \pi^0$ reactions: we then have, for $T=1$,

$$\frac{N(\pi^+ + \pi^-)}{N(\pi^- + \pi^0)} = \frac{18}{1}\frac{[|a_{3/2}|^2 + |a_{1/2}|^2]}{[|a_{3/2}|^2 + 13|a_{1/2}|^2]},$$

$$18 \geq N(\pi^+ + \pi^-)/N(\pi^- + \pi^0) \geq \tfrac{18}{13}. \qquad (4)$$

Experimentally, this number is found to be greater than one.

However, it should be mentioned that this model, using pion-nucleon isobars with pion-pion interaction only in $T=1$ state, allows no fast π^- to be produced in the reaction $\pi^- + p \rightarrow \pi^+ + \pi^- + n$. Correspondingly the data of reference 2 suggest that the $T=1$ pion-pion resonance is active, but that other processes are important as well.

We thank Maurice Goldhaber and R. M. Sternheimer for valuable comment and criticism.

Note: After completion of this work, two preprints have come to our attention which contain similar ideas.[5,6]

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]V. Alles Borelli, S. Bergin, E. Perez-Mendez, and P. Waloschek, Nuovo cimento 14, 211 (1959).

[2]I. Derado, G. Lütjens, and N. Schmitz, Ann. Physik 4, 103 (1959); I. Derado and N. Schmitz, Phys. Rev. (to be published).

[3]R. M. Sternheimer and S. J. Lindenbaum, Phys. Rev. 109, 1723 (1958).

[4]See L. R. Rodberg, Phys. Rev. Letters 3, 58 (1959).

[5]Franco Selleri, CERN Report No. 8511/TH 90 (to be published).

[6]I. Derado, CERN Report No. 8113 (to be published).

ERRATUM

ANALYSIS OF THE EXPERIMENTAL τ'^+ DECAY SPECTRUM AS A TEST OF THE $\Delta T = 1/2$ RULE. S. Björklund, E. L. Koller, and S. Taylor [Phys. Rev. Letters 4, 424 (1960)].

Due to an error, the predicted value of the parameter $a_{\tau'}$ was given as -9.8. Actually, Weinberg's predicted value is $a_{\tau'} \approx -5.4$. Using the latter value, the question of p becoming negative does not arise. At $a_{\tau'} = -5.4$, the likelihood function for the experimental data is down by a factor of ~1.5 from its maximum at $a_{\tau'} = -7.1$. A six-division χ^2 test with $a_{\tau'} = -5.4$ gives $\chi^2 = 4.0$ with a probability of 0.53. Hence the previous conclusion remains unchanged: The present data, as tested by Weinberg's analysis, are consistent with a $\Delta T = 1/2$ rule.

ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

MICROWAVE CONDUCTIVITY OF A PLASMA IN A MAGNETIC FIELD. Donald C. Kelly, Laboratory of Marine Physics, Yale University, New Haven, Connecticut (Received December 28, 1959; revised manuscript received March 16, 1960).

The Boltzmann equation for electrons in a uniform isothermal plasma is solved by expressing the distribution function as a series of orthogonal polynomials in velocity space with time-dependent expansion coefficients. The microwave conductivity is simply related to certain coefficients. Particular attention is devoted to the case in which the plasma is subject to a constant magnetic field and a microwave electric field. By introducing an "effective" electron temperature, convergence is attained for strong as well as weak electric fields. The formulation is particularly suited for problems involving partially ionized gases which contain several species of ions and neutrals. The conductivity of a completely ionized gas is calculated with and without consideration of electron-electron collisions, and the ratio (γ_E) of the two results is graphically illustrated as a function of microwave frequency.

EFFECT OF THE λ TRANSITION ON THE ATOMIC DISTRIBUTION IN LIQUID HELIUM BY NEUTRON DIFFRACTION. D. G. Henshaw, Division of Physics, Atomic Energy of Canada Limited, Chalk River, Ontario, Canada (Received January 18, 1960).

Neutron diffraction patterns for samples of liquid helium at 1.06°K, 2.29°K, and 2.46°K have been measured over the angular range 4° to 64° using 1.06_4A neutrons. The liquid structure factor $i(s)+1$ was deduced for each curve and these show a change which is associated with the λ transition which indicates that the spatial order in the liquid is smaller below the λ point than above. The measurements were transformed to give the radial distribution function $4\pi r^2[\rho(r)-\rho_0]$ from which was deduced the number of neighbors under the first shell of atoms and the nearest distance of approach of two atoms in the liquid. These lie between $8._5$ atoms and $9._8$ atoms and 2.35 A and 2.40 A, respectively.

PRESSURE EFFECT IN THE ATOMIC DISTRIBUTION IN LIQUID HELIUM BY NEUTRON DIFFRACTION. D. G. Henshaw, Division of Physics, Atomic Energy of Canada Limited, Chalk River, Ontario, Canada (Received January 18, 1960).

The angular distribution of 1.06_4A neutrons scattered from liquid helium at temperatures and pressures in the range 1.2°K to 4.24°K and 0 to 51.3 atmospheres for densities up to 0.184 gram/cc has been measured at about 210 equally spaced points in the angular range 5° to 62°. With increasing liquid density, the principal maximum moves to larger angles and increases in height. The liquid structure factors are given for densities of 0.166 gram/cc and 0.184 gram/cc. The density distribution functions are deduced for each of the scattering patterns. A study of these gives 2.27 ± 0.08 A as the nearest distance of approach of two atoms in the liquid. The number of neighbors under the first and second coordinate shells changes from about 6.5 to 8.5 atoms and from about 9 to 5.5 atoms, respectively, for density changes from 0.095 gram/cc to 0.184 gram/cc. The corresponding change in the ratio of their spacings is from 1.47_5 to 1.38 which values are close to $\sqrt{2}$, the theoretical ratio for a close-packed lattice. The analysis shows that the density changes in the liquid and during the solid-liquid transformation cannot be accounted for on the basis of a uniform dilation of a basic structure. The changes in the distribution function caused by pressure are different from those caused by temperature along the normal vapor pressure line.

STRUCTURE OF LIQUID OXYGEN BY NEUTRON DIFFRACTION. D. G. Henshaw, Physics Division, Atomic Energy of Canada Limited, Chalk River, Ontario, Canada (Received February 1, 1960).

The angular distribution of 1.04A neutrons scattered by specimens of liquid oxygen at 90.7°K,

69.0°K, 62.4°K, and 54.7°K has been measured over the angular range 3° to 78°. Pronounced excess scattering at low angles is taken to be magnetic in origin and its form suggests the possible existence of short-range magnetic order in the liquid. The measured distributions corrected for magnetic scattering were transformed to the radial distribution functions $4\pi r^2[\rho(r) - \rho_0]$. An analysis of these gives 1.2_6 atoms and 1.26 A as the number of neighbors in, and the spacing of the diatomic shell, 2.7 A as the nearest distance of approach of 2 atoms in adjacent molecules in the liquid, and about 3.9 A as the spacing of the main density maximum. The total number of atoms in the main density maximum increases from about 16 to 21 atoms for temperature change from 90.7°K to 54.7°K. The possible existence of the O_4 molecule is discussed.

TRANSPORT COEFFICIENTS FROM DISSIPATION IN A CANONICAL ENSEMBLE. Eugene Helfand, Bell Telephone Laboratories, Murray Hill, New Jersey (Received February 5, 1960).

Expressions for the transport coefficients are obtained by studying the average changes with time which occur in members of a canonical ensemble when the initial state of one particle is partially specified. One calculates, both hydrodynamically and statistical mechanically, an appropriately weighted average of the spread of the initial knowledge. In this way expressions analogous to the Einstein equation for the self-diffusion of a Brownian particle are obtained; viz., the viscosity is proportional to a mean-square center of momentum displacement and the thermal conductivity is related to a mean-square center of energy displacement. These expressions may be converted into the integral of autocorrelation function formulas derived previously by others.

THEORY OF THE ROTATIONAL BROWNIAN MOTION OF A FREE RIGID BODY. L. Dale Favro,* Department of Physics, Harvard University, Cambridge, Massachusetts (Received February 10, 1960).

The orientation of a rigid body is specified by the Cayley-Klein parameters. A system of such bodies subject to small random changes in orientation but not subject to any externally applied torque is then considered in some detail. A diffusion equation is derived with certain linear combinations of the Cayley-Klein parameters as independent variables. This equation is expressed in terms of quantum-mechanical angular momentum operators and a Green's function for the equation is obtained as an expansion in angular momentum eigenfunctions. This expansion can be used to calculate averages of various physical quantities in a nonequilibrium distribution of orientations. It may also be used to calculate the spectral density of fluctuating quantities in an equilibrium distribution. Illustrative examples of both of these applications are given.

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DIELECTRIC BREAKDOWN OF LIQUID HELIUM. C. Blank, Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania, and M. H. Edwards, Royal Military College, Kingston, Ontario, Canada (Received February 8, 1960).

The dielectric strength of liquid He^4 under its saturated vapor pressure has been determined from 1.2 to 4.2°K. With $\frac{3}{8}$-inch diameter spherical steel electrodes 0.15 mm apart the average breakdown field, $\bar{E}_b$, is approximately 1 Mv/cm, nearly independent of temperature. At spacings of 0.5 and 1 mm, $\bar{E}_b$ is lower, and appears to decrease with decreasing temperature below 2.5°K. The mechanism of breakdown is not clear, but field emission of electrons from the cathode is probably involved, because the average breakdown voltage $\bar{V}_b$, obtained with a point and plane electrode system when the point is negative, is about half that obtained when the point is positive, both above and below the λ point. Pre-breakdown currents were never detected, and were probably less than 10^{-13} amp, implying an electrical resistivity for liquid helium of 10^{+18} ohm-cm.

POSSIBLE PHASE TRANSITION IN LIQUID He^3. V. J. Emery, Physics Department, University of California, Berkeley, California and A. M. Sessler,* Lawrence Radiation Laboratory, University of California, Berkeley, California (Received February 8, 1960).

A possible phase transition in liquid He^3 has been investigated theoretically by generalizing

the Bardeen, Cooper, and Schrieffer equations for the transition temperature in the manner suggested by Cooper, Mills, and Sessler. The equations are transformed into a form suitable for numerical solution and an expression is given for the transition temperature at which liquid He^3 will change to a highly correlated phase.

Following a suggestion of Mottelson, it is shown that the phase transition is a consequence of the interaction of particles in relative D states.

The predicted value of the transition temperature depends on the assumed form of the effective single-particle potential and the interaction between He^3 atoms. The most important aspects of the single-particle potential are related to the thermodynamic properties of the liquid just above the transition temperature. Two choices of the two-particle interaction, consistent with experiments, yield a second-order transition at a temperature between approximately 0.05°K and 0.1°K. The highly correlated phase should exhibit enhanced fluidity.

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MAGNETIC STRUCTURE OF Mn_4N. W. J. Takei and G. Shirane, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania, and B. C. Frazer, Brookhaven National Laboratory, Upton, New York (Received February 15, 1960).

The magnetic structure of Mn_4N has been determined by neutron diffraction from powders. The cubic unit cell has Mn at the corner and face centers and N at the body center. Standard diffraction techniques led to four possible models and it was necessary to perform polarized neutron beam experiments to resolve this ambiguity. The structure is ferrimagnetic with a corner moment of $3.5\mu_B$ antiparallel to the three face-center moments of $0.7_7\mu_B$.

GYROMAGNETIC RATIOS OF Fe AND Ni. G. G. Scott, Research Laboratories, General Motors Corporation, Warren, Michigan (Received February 4, 1960).

It has been shown that the previously observed decrease in the values of g' for weakly magnetized specimens of Fe and Ni was caused by a systematic error in the measurement of magnetic moment. Recent experiments on these two metals indicate g' values of 1.919 ± 0.002 for Fe and 1.835 ± 0.002 for Ni.

NUCLEAR MAGNETIC RESONANCE IN $KMnF_3$. R. G. Shulman and K. Knox, Bell Telephone Laboratories, Murray Hill, New Jersey (Received February 12, 1960).

The nuclear magnetic resonance of fluorine in $KMnF_3$ has been studied and hyperfine interactions between the fluorine nucleus and the magnetic electrons measured. The values are $A_S = (16.26 \pm 0.4) \times 10^{-4}$ cm^{-1} and $(A_\sigma - A_\pi) = (0.17 \pm 0.1) \times 10^{-4}$ cm^{-1}. These values correspond to $(0.50 \pm 0.02)\%$ $2s$ and $(0.18 \pm 0.1)\%$ $2p$ character for the unpaired electron. The implications of these results in terms of the σ and π bonding in this compound are discussed. By a comparison with hyperfine interactions measured in recent paramagnetic resonance studies of $KMgF_3:Mn^{+2}$, the amount of distortion in the mixed crystal is estimated. An antiferromagnetic transition is observed at 88.5°K.

ETCH PITS ON DENDRITIC GERMANIUM: A CLARIFICATION. P. J. Holmes, Research Laboratory, Associated Electrical Industries, Aldermaston Court, Aldermaston, England (Received February 1, 1960).

Previous reports by Billig and Holmes of the orientations of etch pits on the main faces of germanium dendrites are not at first sight consistent with those recently reported by Bennett and Longini. It is shown that this discrepancy arose because the orientation of equilateral pits on {111} surfaces depends on the etchant used. A check on the earlier work confirms that there is in fact no contradiction: ferricyanide and WAg etches produce pits which point upwards on "perfect" faces grown in a "G direction," while superoxol and similar etchants give pits pointing downwards.

PHONON BROADENING OF IMPURITY LINES. E. O. Kane, Hughes Products, Newport Beach, California (Received January 27, 1960).

The theory of line broadening given by Lax is found not to apply to shallow impurities in Ge and Si. Observed broadening may be instrumental at low temperatures and may be due to

lifetime broadening through phonon interaction at nitrogen temperature. Calculations suggest that p-type spin resonance in germanium and silicon may be observable.

PARAMAGNETIC RESONANCE SPECTRUM OF MANGANESE IN CORUNDUM. W. Low and J. T. Suss,* Department of Physics, The Hebrew University, Jerusalem, Israel (Received February 3, 1960).

The paramagnetic resonance spectrum of Mn^{2+} in Al_2O_3 was measured at 3 cm. The spectrum can be expressed in a spin Hamiltonian with trigonal symmetry with the following parameters: $g_{\parallel}=2.0017\pm0.001$, $g_{\perp}=2.000\pm0.002$, $D=+194.2\pm1$, $a-F=+21.9\pm0.6$, $A=-79.6\pm0.5$, $B=-78.8\pm0.8$ in units of 10^{-4} cm^{-1}.

*On leave from the Atomic Energy Commission of Israel.

ANGULAR DISTRIBUTION OF SECONDARY ELECTRONS FROM (100) FACES OF COPPER AND NICKEL. Jay Burns, Department of Physics and Chicago Midway Laboratories, The University of Chicago, Chicago, Illinois (Received February 8, 1960).

The angular distributions of secondary electrons from (001) faces of copper and nickel single crystals have been measured for secondaries in four energy ranges (0-10 ev, 10-20 ev, 20-40 ev, and 40-90 ev) for primary electron energies of 250, 500, and 800 ev. Fine structure was observed which consisted of weak peaks in the angular distribution superimposed on a background having approximately a cosine distribution. After making corrections for the refraction of secondaries at the surface of the crystal, the internal angular distribution peaks fall along principal low-index directions in the crystal as suggested in the quantum mechanical collision theories of Wooldridge and of Dekker and van der Ziel. The positions, intensities, and widths of the peaks cannot be accounted for in terms of diffraction of the internal secondaries. The observed peaks are believed to be secondaries produced in the initial collision between the primary electron and a lattice electron of the crystal, enough of these secondaries having escaped the crystal without further collisions to make their observation possible. Details of the angular distribution are in agreement with collision theory based on a screened Coulomb interaction with a velocity-dependent screening length. The velocity dependence of the screening coefficient in the screened Coulomb interaction leads to a sharp drop in the inelastic cross section for energy transfers larger than the plasma excitation energy, and it also leads to increased probability for collisions in which the primary suffers only small deflections. The role of the band structure of the crystal in determining the features of the collision is discussed. In Cu and Ni the vacuum level of potential lies in the second Brillouin zone, so only interzone (Umklapp) transitions can lead to secondary electron emission from these metals. Surface refraction is treated in terms of a velocity-dependent refractive index, and the experiment offers a means of determining the velocity dependence of the index. Experimental procedures and precautions required to observe the angular distribution fine structure are discussed.

TEMPERATURE DEPENDENCE OF OPTICAL BLEACHING OF KCl CRYSTALS NEAR 0°C. W. E. Bron and A. S. Nowick, International Business Machines Research Center, Yorktown Heights, New York (Received February 12, 1960).

Further insight has been sought into the mechanism of optical bleaching in the vicinity of room temperature by studying the temperature dependence of the bleaching curves for KCl crystals which had been initially irradiated with hard (filtered) x rays. The absorptions at the maxima of the F, M, R, and R_2 bands were observed to change during illumination with F light and to be strongly temperature dependent in the range of -30°C to +10°C, whereas the absorption at the V_3 band was essentially unchanged. For comparison an analysis was made of the data of Petroff on the early growth of the M band during bleaching with F light in additively colored KCl crystals. In this case a unique activation energy, ϵ, of 0.35 ± 0.05 ev and a number of defect jumps $N_j\approx10^{10}$ are indicated by the data. It appears that the bleaching curves of the x-irradiated samples are composed of a temperature-independent and a temperature-dependent part. The temperature-dependent part probably is the same as that responsible for bleaching in additively colored crystals. The above results for ϵ and n_j suggest that the temperature-dependent bleaching process results from the trapping of photo-

electrons at vacancy clusters which are formed during bleaching through the migration of mobile defects, possibly vacancy pairs. This conclusion is supported by the observations by others that the α band is not observed during bleaching at room temperature.

REDUCTION IN COERCIVE FORCE CAUSED BY A CERTAIN TYPE OF IMPERFECTION. A. Aharoni, Department of Electronics, The Weizmann Institute of Science, Rehovot, Israel (Received February 1, 1960).

As a first approach to the study of the dependence of the coercive force on imperfections in materials which have high magnetocrystalline anisotropy, the following one-dimensional model is treated. A material which is infinite in all directions has an infinite slab of finite width in which the anisotropy is 0. The coercive force is calculated as a function of the slab width. It is found that for relatively small widths there is a considerable reduction in the coercive force with respect to perfect material, but reduction saturates rapidly so that it is never by more than a factor of 4.

ACTIVATION ENERGY FOR THE SURFACE MIGRATION OF TUNGSTEN IN THE PRESENCE OF A HIGH ELECTRIC FIELD. Philip C. Bettler and Francis M. Charbonnier, Linfield Research Institute, McMinnville, Oregon (Received February 5, 1960; revised manuscript received April 4, 1960).

An activation energy for the surface migration of tungsten atoms on the tungsten crystal lattice structure and under the influence of a high electric field has been measured using field emission techniques. The initially hemispherical field-emitter tip surface deforms into a polyhedral shape in a process known as buildup, when the emitter is heated in the presence of large electrostatic forces. Buildup proceeds in a regular and reproducible manner; certain stages of buildup can be identified by characteristic changes in both the field emission patterns and the current vs time characteristics of the emitter. An activation energy of 2.44 ± 0.05 ev/atom was determined, from the measured values of the time required to achieve a given degree of buildup at various operating temperatures. This value may be compared with the value of 3.14 ev/atom determined from the rate at which the tip of a heated tungsten emitter recedes in the absence of an electric field. Explanations for the difference are presented, involving two distinct factors: (1) a reduction in activation energy, through the effect of polarization of the surface atoms by the electrostatic field, by an amount which was determined in a special experiment; and (2) an inherent difference which remains after allowance has been made for the field effect. The latter is ascribed to the difference in the paths of migration in the two cases whereby, for the conditions existing in this experiment, the activation energy measured is that corresponding to migration primarily over the low-index (100), (110), and (211) planes. A value of 2.79 ± 0.08 ev/atom is obtained after correction for the field.

SURFACE-DEPENDENT $1/f$ NOISE IN GERMANIUM. A. U. MacRae* and H. Levinstein, Physics Department, Syracuse University, Syracuse, New York (Received February 4, 1960).

The surface characteristics of $1/f$ noise have been investigated by using field-effect techniques on 100-micron thick single-crystal germanium filaments. The $1/f$ noise is independent of the surface potential when an accumulation layer is on the surface but increases rapidly as the surface conductivity gradually becomes inverted with respect to the bulk. No $1/f$ noise is observed due to charge transfer between the bulk and the slow surface states. An increase in the $1/f$ noise associated with the inversion layer occurs when the temperature of the germanium is decreased. The magnitude of the $1/f$ noise depends on the ambient, increasing as the slow state relaxation time decreases. An investigation of the relaxation processes associated with the charge transfer between the bulk and the slow surface states after the application of a dc electric field to the field-effect electrode reveals a $1/f$ noise relaxation which is independent of the mode of the conductivity relaxation. The noise relaxes back to its original value with a logarithmic time dependence which is characteristic of a $1/\tau$ distribution in time constants and the conductance decays with a combination of exponential and logarithmic terms, depending on the surface conditions.

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APPROXIMATE WAVE FUNCTIONS FOR ATOMIC Be. R. E. Watson,* Solid-State and Molecular Theory Group, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received February 5, 1960).

A configuration interaction calculation, involving thirty-seven configurations and including the $(1s)^2(2s)^2$ Hartree-Fock function, has been done for the ground state of atomic Be. Approximately ninety percent of the correlation energy has been incorporated into the final total energy. The results indicate that the correlation energy is associated with two effects, namely that of the "correlation hole" as has been observed for He and that of "orbital degeneracy" (which does not appear in the two-electron He case). The former effect is best handled by the Hylleraas approach and the latter by the configuration interaction method, and the results suggests that an admixture of the two methods would lead to the most rapidly convergent description of the exact four-electron wave function. The errors introduced by handling "high-lying" configurations by second order perturbation theory rather than by exact configuration interaction are also investigated.

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UPPER BOUNDS ON ELECTRON-ATOMIC HYDROGEN SCATTERING LENGTHS. Leonard Rosenberg, Larry Spruch, and Thomas F. O'Malley, Physics Department, New York University, New York, New York (Received February 1, 1960).

Recently developed variational techniques for determining upper bounds on scattering lengths are applied to singlet and triplet scattering of zero-energy electrons by atomic hydrogen. The results obtained are not only rigorous but are in fact somewhat lower and therefore somewhat better than those previously obtained by variational methods. We find that the triplet and singlet scattering lengths, A_T and A_S, respectively, satisfy the inequalities $A_T \leqq 1.91a_0$ and $A_S \leqq 6.23a_0$, where a_0 is the Bohr radius. The only assumptions involved in the deduction of these results are that there be no bound triplet state and one and only one bound singlet state. The singlet trial function determined during the course of the calculation generates a singlet effective range, r_{0S}, of about $2.7a_0$. The triplet trial functions which were obtained were not sufficiently accurate to be useful in a determination of the triplet effective range, r_{0T}.

EFFECT OF MUTUAL DISTORTION ON PHASE SHIFTS OF COLLIDING SYSTEMS. Ian C. Percival,* Stanford Research Institute, Menlo Park, California (Received February 12, 1960).

The box variational principle for scattering phase shifts is extended to one-channel collisions of arbitrary angular momentum without exchange, and also to systems with many degrees of freedom, when the energies are nonrelativistic and insufficient to produce inelastic collisions. Under reasonable assumptions of continuity it is proved that the commonly used one-state and many-state approximations always reduce the scattering phase shift from its correct value, so long as no further approximations have to be made, and thus provide lower bounds to the exact scattering phase shift. The distorted-wave approximation is an example. The inclusion of more states into a many-state approximation never makes the estimated phase any worse, and generally improves it. Mutual distortion of colliding systems never reduces the phase shift and generally increases it, thus producing an effective attraction between the systems.

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BORN CROSS SECTIONS FOR INELASTIC SCATTERING OF ELECTRONS BY HYDROGEN ATOMS. I. 3s, 3p, 3d STATES. Gerard C. McCoyd, S. N. Milford, and John J. Wahl, St. John's University, Jamaica, New York (Received January 20, 1960).

Born cross sections of all $n = 3$ to $n = 4$ transitions are calculated at ten incident electron energy values in the range 0.67 - 1400 ev, and those of strong optically allowed $n = 3$ to $n = 5$ transitions are calculated at five incident electron energy values in the range 1 - 10 000 ev. The cross sections obtained are much larger than for comparable transitions from the ground state, and the cross sections for transitions which are optically allowed and in which n and l change in the same sense are larger than those for other transitions. For all strong optically allowed transitions the Bethe (dipole) approximations to the Born cross sections are calculated and

comparison shows that the Bethe formula gives a good fit to the Born approximation down to relatively low energies (≈ 10 ev).

BORN CROSS SECTIONS FOR INELASTIC SCATTERING OF ELECTRONS BY HYDROGEN ATOMS. II. $4s$, $4p$, $4d$, $4f$ STATES. Leonard Fisher, S. N. Milford, and Frank R. Pomilla, St. John's University, Jamaica, New York (Received January 20, 1960).

The Born total cross sections are calculated for the inelastic scattering of electrons by hydrogen atoms for the strong optically allowed transitions from $n=4$ to $n'=5$. The nine incident energies considered range from 0.546 ev to 1361 ev. In addition, the $4s$ to $6p$ and $4f$ to $6g$ transitions are considered. Bethe (multipole) cross sections are also calculated and found to reproduce the Born results down to low energies.

HYPERFINE STRUCTURE OF THE 6^3P_2 STATE OF $_{80}Hg^{199}$ AND $_{80}Hg^{201}$. PROPERTIES OF METASTABLE STATES OF MERCURY. Mark N. McDermott* and William L. Lichten,† Columbia Radiation Laboratory, Columbia University, New York, New York (Received January 29, 1960).

The hyperfine structures of the metastable 6^3P_2 state of Hg^{199} and of Hg^{201} have been measured by means of the atomic-beam magnetic resonance technique. The present experiment is the first one for which the electron bombardment method has been used in the production of a narrowly collimated beam of metastable atoms. The beam was detected by surface ejection of electrons from an alkali metal surface. The zero magnetic field intervals $f(F \rightarrow F')$ are: for Hg^{199}, $f(5/2 \rightarrow 3/2) = 22\,666.559(5)$ Mc/sec; and for Hg^{201}, $f(7/2 \rightarrow 5/2) = 11\,382.6288(8)$ Mc/sec, $f(5/2 \rightarrow 3/2) = 8629.5218(5)$ Mc/sec, and $f(3/2 \rightarrow 1/2) = 5377.4918(20)$ Mc/sec. The values of the quadrupole and octupole moments of Hg^{201} are, without polarization corrections, $Q = 0.50(4) \times 10^{-24}$ cm^2 and $\Omega = 0.13$ nuclear magneton barn. The hyperfine structure anomaly for the two isotopes due to the s electron alone is $\Delta(s_{1/2}) = -0.1728(12)\%$ in disagreement with the predictions of the single-particle model.

The g_J values for the $^3P_2$ state and the $(5d^9 6s^2 6p)^3D_3$ state were found to be $g_J(^3P_2) = 1.50099(10)$ and $g_J(^3D_3) = 1.0867(5)$. The value of $J=3$ for the $^3D_3$ state was confirmed. A new technique for obtaining excitation functions is discussed.

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OBSERVATION OF π STARK COMPONENTS IN MICROWAVE SPECTROSCOPY: PRECISION MEASUREMENTS ON HCN. Brojo N. Bhattacharya and Walter Gordy, Department of Physics, Duke University, Durham, North Carolina (Received February 4, 1960).

A parallel plate Stark cell has been designed and constructed for the millimeter wave region of the spectrum. With the cell both π and σ Stark components can be observed. High-precision Stark effect measurements have been made on the π and σ components of the $J=0 \rightarrow 1$ transition of HCN^{14}. From these the electric dipole moment of HCN in the ground vibrational state has been calculated to be 2.985 ± 0.005 Debye units.

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RADIOACTIVE DECAY OF Tm^{166}. R. G. Wilson and M. L. Pool, Department of Physics and Astronomy, The Ohio State University, Columbus, Ohio (Received January 25, 1960).

Erbium oxide enriched to 72.9% in the 166 mass number was irradiated with 6-Mev protons. An activity decaying by electron capture and positron emission with a half-life of 7.69 ± 0.05 hours was produced by a (p,n) reaction and its assignment to Tm^{166} confirmed. The observed activity consists of the K x ray of erbium, gamma rays with energies of 81, 184, 289, 405, 460, 598, 674, 694, 707, 759, 782, 788, 878, 1052, 1179 1276, 1351, 1589, 1874, and 2058 kev, annihilation radiation, and particle radiation with an end-point energy of 2090 ± 40 kev. Gamma-gamma coincidence measurements and consideration of the energies and relative numbers of the observed radiations have led to the assignment or confirmation of energy levels at 81 (2+), 265 (4+), 554 (6+), 788 (2+), 863 (3+), 959 (4+), 1248 (2), 1317 (5), 1462 (0+), 1547 (3+), 1701 (4+), 1894 (5+), 2139 (3), and 2168 (0) kev in Er^{166}. The 2139-kev level is highly populated by electron capture and the positron transitions occur

to the 265 (4+)-kev level. The positions of the observed radiations and the branching ratios of electron capture are shown in a proposed energy level scheme.

PROTON-PROTON SCATTERING AT 68 Mev. D. E. Young* and L. H. Johnston, University of Minnesota, Minneapolis, Minnesota (Received February 4, 1960).

Differential cross sections have been measured for the scattering of 68.3-Mev protons by hydrogen gas at 26 laboratory angles from 5° to 50°. The angular resolution is $\pm 1/2°$ at small angles, and the estimated absolute probable errors are $\pm 0.9\%$ except at the smallest angles. The interference minimum of 5.19 millibarns occurs at 16.3° c.m. The cross section then rises to a maximum of 6.33 mb at 34° and falls to 6.16 mb at 90°.

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BETA-DECAY THEORY AND THE SPECTRUM OF Rb^{87}. M. A. Preston, G. H. Keech, and J. M. Pearson,* Department of Physics, McMaster University, Hamilton, Ontario, Canada (Received January 11, 1960).

The shape of the third forbidden β spectrum of Rb^{87} has been analyzed. It has been found to be consistent with (i) a mixture of vector and axial vector interactions, (ii) the same value and sign of the ratio g_A/g_V as found for the neutron, and (iii) a shell-model evaluation of the nuclear matrix elements.

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DECAY OF Ga^{66} AND Cu^{66}. Arthur Schwarzschild and Lee Grodzins,* Brookhaven National Laboratory, Upton, New York (Received February 11, 1960).

The 9.5-hour decay of Ga^{66} and the 5.1-minute decay of Cu^{66} have been studied by a variety of techniques including gamma-ray spectroscopy, internal conversion measurements, and angular correlation studies. All but one of the 18 gamma rays observed have been ordered into a structure of 11 levels and the spins and parities of all but one of these levels have been determined. It is shown that Ga^{66} has spin zero and even parity and that its 4.166-Mev β spectrum is a pure Fermi transition. The energy of the internal conversion line of the 4.300 ± 0.005 Mev transition was measured with great care and this line may be useful for spectrometer calibration. The second excited state of Zn^{66}, at 1.875 Mev, has 2+ spin and parity. The stopover transition from this state contains at least 10% $M1$ radiation; the stopover to crossover ratio is greater than 100 to 1.

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BRANCHING OF TRANSITIONS IN SOME MIRROR NUCLEI. W. L. Talbert, Jr.* and M. G. Stewart, Institute for Atomic Research and Department of Physics, Iowa State University, Ames, Iowa (Received January 25, 1960).

The possibility of branching in the decays of Na^{21}, Mg^{23}, Al^{25}, Si^{27}, S^{31}, and Ca^{39} has been investigated using NaI(Tl) scintillation detectors. The nuclear gamma rays emitted as a result of branching transitions were detected in coincidence with the accompanying positron annihilation radiation. Branching was found to the first excited states of the daughter nuclei in the decays of Na^{21}, Mg^{23}, and S^{31}, with intensities (compared to the total decay) of 2.2, 9.1, and 1.1%, respectively. The decays of Al^{25}, Si^{27}, and Ca^{39} were found to have no detectable branching to the lower excited states of the daughter nuclei, and upper limits of less than one percent were placed on the branching ratios for such branches. The lack of branching in the decay of Al^{25} to the 0.98-Mev level of Mg^{25} favors a unified model description for the nuclear states involved.

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RECOIL STUDIES OF HEAVY-ELEMENT NUCLEAR REACTIONS. I. Paul F. Donovan,* B. G. Harvey, and W. H. Wade,† Lawrence Radiation Laboratory, University of California, Berkeley, California (Received February 15, 1960).

Techniques have been developed which permit the accurate measurement of angular distributions of recoil nuclei formed in nuclear reactions. The angular distributions of recoils from the

reactions $Bi^{209}(\alpha, 3n)At^{210}$, $Bi^{209}(\alpha, 4n)At^{209}$, and $Bi^{209}(d, 3n)Po^{208}$ are consistent with a reaction mechanism involving the formation of a compound nucleus and subsequent isotropic evaporation of the neutrons, as shown by comparison with Monte Carlo calculations based on an isotropic evaporation model.

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RECOIL STUDIES OF HEAVY-ELEMENT NUCLEAR REACTIONS. II. B. G. Harvey, W. H. Wade,* and Paul F. Donovan,† Lawrence Radiation Laboratory, Berkeley, California (Received February 15, 1960).

Angular distributions and ranges of recoils from the reactions $Bi^{209}(\alpha, 2n)At^{211}$ and $Cm^{244}(\alpha, 2n)Cf^{246}$ have been measured. At helium ion energies higher than about 10 Mev above the Q values of these reactions, the results are consistent with a reaction mechanism involving the emission of one or both neutrons in the forward hemisphere.

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PERTURBATION THEORY APPLIED TO THE NUCLEAR MANY-BODY PROBLEM. J. S. Levinger, M. Razavy, O. Rojo, and N. Webre, Louisiana State University, Baton Rouge, Louisiana (Received December 17, 1959).

Perturbation theory is applied to infinite nuclear matter at the observed density for a well-behaved two-body potential, containing a tensor force. We find that a tensor force can contribute as much as 10 Mev/particle to the binding energy in second order. Perturbation theory is then modified to include the pseudopotential treatment of an infinite repulsive core. We give a detailed derivation of the DeDominicis-Martin and Huang-Yang result for a pure repulsive core. We obtain an expansion jointly in powers of the strength of the attractive potential, and in the range of the core. We find the second order contributions to the binding energy for several potentials combining an infinite repulsive core with an attractive potential. For each case considered, the second order terms are large (absolute value about 20 Mev/particle).

ACCURATE METHOD FOR MEASURING INTERNAL CONVERSION COEFFICIENTS. D. C. Lu, Institute for Atomic Research and Department of Physics, Iowa State University, Ames, Iowa (Received February 10, 1960).

To reveal the effect on internal conversion due to the nuclear structure and extension, measurements are needed which have higher accuracy than is attainable from currently used methods. This article describes how, under certain conditions, the absolute value of the total internal conversion coefficient can be measured to $\pm 0.5\%$ by the use of a large NaI(Tl) detector with a thin well-type window. Complications encountered in the comparison between experimental and computed values are mentioned.

EXCITATION FUNCTION FOR $Zn^{64}(n, 2n)Zn^{63}$. D. R. Koehler and W. L. Alford, Research Laboratory, Ordnance Missile Laboratories Division, Army Rocket and Guided Missile Agency, Redstone Arsenal, Alabama (Received February 8, 1960).

The excitation function for $Zn^{64}(n, 2n)Zn^{63}$ has been measured for neutron energies from 12.2 to 18.1 Mev by an activation method. An absolute cross section has been obtained by using the previously measured value of 167 ± 11 mb at 14.4 Mev. Above threshold, the cross section is found to increase rapidly with neutron energy, reaching a value of 337 mb at 18.1 Mev. A cross-section curve computed on the basis of statistical theory is shown for comparison.

GAMMA RADIATIONS OF Na^{23} AND Ne^{20}. T. H. Kruse, R. D. Bent,* and L. J. Lidofsky, Columbia University, New York, New York (Received January 18, 1960).

Na^{23} and Ne^{20} gamma rays were observed from the proton bombardment at various energies of thin evaporated Na and NaI targets and of a natural Ne gas target. Gamma rays involving Na^{23} states up to 4 Mev and Ne^{20} states up to 5 Mev were observed and decay schemes and branching ratios obtained. The 2.08- and 2.70-Mev states

of Na^{23} are probably $7/2^+$ and $9/2^+$, respectively. Limitations on spin and parity values are given for other states. The results obtained for Na^{23} are consistent with the results of a strong-coupling collective calculation. The 4.97-Mev state of Ne^{20} has an upper limit for the ground-state branch of 4%. An upper limit of 9% is placed on the ground-state branch of the 4.2-Mev state of Ne^{20}. Gamma rays from the $F^{19}(d,n\gamma)Ne^{20}$ reaction were observed with energies of 11.4, 10.67, 10.16, 9.37, 8.37, and 7.36 Mev.

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ELASTIC SCATTERING OF DEUTERONS BY He^4. J. L. Gammel, B. J. Hill,* and R. M. Thaler,† Los Alamos Scientific Laboratory, Los Alamos, New Mexico (Received February 5, 1960).

A model of the $d+He^4$ interaction is developed and compared to the data on the ground state of Li^6 and the $d+He^4$ elastic scattering data up to 4.5 Mev (deuteron laboratory energy). New phase shift analyses of the 8- and 10.3-Mev elastic scattering data are made, and quantities relevant to the production or analysis of beams of polarized deuterons are calculated.

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POSSIBILITY OF A FISSION CHAIN REACTION IN SUPERNOVA TYPE I. Peter Fong, Physics Department, Utica College of Syracuse University, Utica, New York, and Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California (Received February 10, 1960).

The possibility of a fission chain reaction is discussed for the purpose of explaining the discrepancy between the observed light intensity of supernova type I in its decaying stage and the amount of energy available from the spontaneous fission of Cf^{254}. A convergent fission chain reaction would make the energy output many times larger while at the same time it would keep the half-life of the light intensity curve the same as that of the spontaneous fission of Cf^{254}. However, the necessary conditions for this mechanism to contribute appreciably to the energy source do not seem to exist in supernova type I; other mechanisms are required to explain the discrepancy.

MEASUREMENT OF THE CIRCULAR POLARIZATION OF RESONANCE-SCATTERED GAMMA RAYS FOLLOWING THE ELECTRON CAPTURE OF Se^{75}. F. Boehm and C. J. Gallagher, Jr.,* California Institute of Technology, Pasadena, California (Received February 8, 1960).

The circular polarization of the 265-kev γ rays following the mixed Gamow-Teller and Fermi electron-capture decay of Se^{75} into As^{75} has been measured. The neutrino momentum was fixed with the help of a resonance scattering process. From the experimentally determined degree of right-hand circular polarization of -0.21 ± 0.15 it was concluded that the sign of the Gamow-Teller to Fermi matrix-element ratio in this beta decay is negative.

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ENERGY GAP IN NUCLEAR MATTER. V. J. Emery, Physics Department, University of California, Berkeley, California, and A. M. Sessler,* Lawrence Radiation Laboratory, University of California, Berkeley, California (Received February 8, 1960).

The magnitude of the energy gap in nuclear matter associated with a highly correlated ground state of the type believed to be important in the theory of superconductivity has been evaluated theoretically. The integral equation of Cooper, Mills, and Sessler is linearized and transformed into a form suitable for numerical solution. The energy gap, calculated by using an appropriate single-particle potential and the Gammel-Thaler two-body potential, is found to be a very strong function of the density of nuclear matter, and of the effective mass at the Fermi surface. It is concluded that the magnitude of the energy gap for nuclear matter should not be compared directly with experimental values for finite nuclei, although the results suggest that if the theory is extended to apply to finite nuclei it probably would be in agreement with experiment.

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TWO-NUCLEON STRIPPING PROCESS. M. el Nadi,* Sloane Physics Laboratory, Yale University, New Haven, Connecticut (Received December 28, 1959).

A new expression is derived for the differential cross section of processes in which two nucleons are captured from an incident alpha particle or similar projectiles. The formula derived is compared with a similar one previously obtained together with some experimental data on the $O^{16}(d, \alpha)N^{14}$ reaction. Fairly good agreement is observed.

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DECAY OF μ^- MESONS BOUND IN THE K SHELL OF LIGHT NUCLEI. H. Überall, Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received February 15, 1960).

For μ^- mesons bound in the K shell of light nuclei of atomic number Z, we calculate the decay electron spectrum accurately up to the first power in Z, both for point and extended nuclei. The decay rate is evaluated accurately up to the second power for point nuclei. Our results for the spectrum show the Doppler smearing of its upper end as obtained previously, and demonstrate the small effect of the nuclear extension. The decay rate is obtained as a monotonically decreasing function of Z, and we cannot explain recent experiments which show a maximum of the decay rate around $Z \sim 26$. We also find that the decay rate in second order decreases much more slowly with Z than what would be obtained from a phase space consideration alone.

DEPOLARIZATION AND TIME REVERSAL IN p-p SCATTERING AT 142 Mev. C. F. Hwang, T. R. Ophel,* E. H. Thorndike, and Richard Wilson, Cyclotron Laboratory, Harvard University, Cambridge, Massachusetts (Received February 12, 1960).

The depolarization in proton-proton scattering at 142 Mev was measured at angles from 6° to 40° in the laboratory system.

The measurements were made by scattering a 67% polarized proton beam first off a liquid hydrogen target, then off a carbon (or lithium) analyzer. The scattered protons were detected by plastic scintillation counters, and the asymmetries from the last scattering were measured at each hydrogen scattering angle. The angular dependence of depolarization determined in this work was similar to that measured at 315 Mev by other workers. The data disagree with other measurements at 143 Mev.

By measuring on both sides of the beam, the polarization in scattering is determined and compared with asymmetry in scattering from a polarized beam. Their equality confirms time-reversal invariance in the proton-proton interaction.

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PROTON-PROTON DEPOLARIZATION AT 98 Mev. E. H. Thorndike and T. R. Ophel,* Cyclotron Laboratory, Harvard University, Cambridge, Massachusetts (Received February 12, 1960).

The triple scattering depolarization parameter D for proton-proton scattering has been measured at 98 Mev, as follows: 10° (lab), 0.00 ± 0.08; 15°, 0.00 ± 0.07; 20°, 0.00 ± 0.08; 25°, -0.12 ± 0.10; 30°, -0.11 ± 0.16.

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DEPENDENCE ON ATOMIC NUMBER OF THE NUCLEAR PHOTOEFFECT AT HIGH ENERGIES. P. C. Stein,* A. C. Odian,† A. Wattenberg,† and R. Weinstein,‡ Physics Department and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received February 1, 1960).

A measurement was made of the number of neutron-proton coincidences observed when 320-Mev bremsstrahlung bombarded D, Li, Be, C, O, Al, Ti, Cu, Sn, and Pb. If one normalizes the data for the number of neutron-proton pairs in a nucleus (i.e., by dividing by NZ/A) it is found that the observed coincidences decrease as A increases.

It is possible to account quantitatively for this A dependence by correcting for the probability that two nucleons will escape from inside a nucleus without either having a collision. The probability of escape is a function of the nuclear radius, R, and the mean free path, λ, in nuclear matter. For medium-weight elements the observed neutron-proton pairs are produced with

a cross section given by

$$\sigma_{Z,A}(\text{coincidences}) \cong 3.0(NZ/A)\sigma_D P(2R/\lambda),$$

where σ_D is the cross section for the photodisintegration of the deuteron and where $P(2R/\lambda)$ is the probability-of-escape factor. For two nucleons emitted at 180° the form of $P(x)$ is

$$P(x) = (3/x^3)[2 - e^{-x}(x^2 + 2x + 2)].$$

The formula for the cross sections is shown to be what one would expect if the fundamental mechanism in complex nuclei is the same as that suggested by Wilson for the photodisintegration of the deuteron. The constant, 3.0, depends on the cube of a neutron-proton pair interaction distance. A less naive treatment also involves a nucleon pair correlation function.

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CORRECTIONS TO THE IMPULSE APPROXIMATION FOR PHOTON-DEUTERON SCATTERING. R. L. Schult and R. H. Capps, Laboratory of Nuclear Studies, Cornell University, Ithaca, New York (Received February 1, 1960).

The validity of the impulse approximation for the scattering of 50-120 Mev photons from deuterons is investigated by the use of forward scattering dispersion relations. The only significant deviation from the impulse approximation found is in the spin-independent amplitude. Most of this deviation is shown to be the result of the exchange part of the neutron-proton potential. The exchange force is known to increase the electric dipole photodisintegration cross section; it is shown that the exchange force also increases the electric dipole elastic scattering cross section by about 10-20%.

THEORY OF ALLOWED AND FORBIDDEN TRANSITIONS IN MUON CAPTURE REACTIONS. II. Masato Morita, Columbia University, New York, New York, and Daniel Greenberg, Barnard College, Columbia University, New York, New York (Received February 2, 1960).

The general formalism of the first paper in this series is applied to the calculation of the angular distribution of the recoils in muon capture. Only the unique nth forbidden transitions [spin change $0 \to J$, parity change $(-)^{J+1}$] are considered. As an example the special case of C^{12} is discussed. The angular distribution of the recoils depends strongly on the strength of the induced pseudoscalar interaction, but is rather insensitive to the assumption of conserved vector current.

VELOCITY DEPENDENCE OF THE BUBBLE DENSITY FOR CHARGED PARTICLE TRACKS IN LIQUID HYDROGEN. V. P. Kenney, Brookhaven National Laboratory, Upton, New York and the University of Kentucky, Lexington, Kentucky (Received December 21, 1959; revised manuscript received March 31, 1960).

Bubble densities of tracks of 635-Mev/c protons and pions in a liquid hydrogen bubble chamber operated at 26.5°K, 62 psig have been determined from measurements of the distribution in spacing of the individual bubbles. The velocity dependence of the bubble density has been obtained by fitting the bubble densities observed to the expression $m = A/\beta^b$ by the least-squares method, yielding the values $A = 8.64$ bubbles/cm, and exponent $b = 1.935 \pm 0.077$. The constant A is a function of the temperature of the liquid hydrogen, varying ~30% per 0.1°K. If the number of bubbles per unit track length observed is correlated with the rate of delta-ray formation, it would appear that an energy of the order of 400 ev is necessary for bubble nucleation in liquid hydrogen.

YIELD OF NEUTRONS PER INTERACTION IN U, Pb, W, and Sn BY PROTONS OF SIX ENERGIES BETWEEN 250 AND 900 Mev SELECTED FROM COSMIC RADIATION. M. Bercovitch, H. Carmichael, G. C. Hanna, and E. P. Hincks,* Physics Division, Atomic Energy of Canada Limited, Chalk River, Canada (Received February 15, 1960).

The production of low-energy neutrons in U, Pb, W, and Sn, by protons of six selected energies between 250 and 900 Mev has been measured using cosmic radiation as a proton source. The protons were selected and their energy measured by a vertical counter telescope containing three Čerenkov detectors which employed liquid nitrogen, water, and Plexiglas as radiating media. The protons interacted in 22 and 44 g cm^{-2} thick slabs of the target elements, and the

neutrons produced were detected in a 4-foot cubic paraffin moderator, $B^{10}F_3$ counter assembly placed below the proton selecting telescope. The principal body of data was obtained at 3260-m altitude; a series of runs at 150 m was made to check the high-altitude data for muon contamination of the selected protons.

The proton-gated neutron rates for the various targets were converted to mean neutron multiplicities per interaction using (*a*) the efficiency of the neutron detector as measured using calibrated Pu^{240} spontaneous fission and Ra-α-Be neutron sources, (*b*) the interaction cross sections of Chen, Leavitt, and Shapiro. The mean multiplicities per interaction range from 5.8 ± 1.0 for 300-Mev protons on 33 g cm^{-2} thick Sn, to 26.7 ± 4.2 for 820-Mev protons on 44 g cm^{-2} thick U. The multiplicities predicted from the Monte Carlo nucleon cascade calculation of Metropolis et al., and the Monte Carlo evaporation calculation of Dostrovsky et al., are in agreement with the measurements when secondary neutron production in the thick targets is taken into account.

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SMALL-ANGLE PROTON-PROTON SCATTERING AT 435 Mev. S. K. Kao, H. Horstman, and C. W. Hinman, Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received January 6, 1960).

This paper presents the results of measurements of the scattering cross sections of protons by protons in the angular range 5°-20° (center of mass). An ionization chamber was used to measure the direct beam and the scattered protons were detected by means of photographic plates. The results are substantially in agreement with other work in this range of energy and angle although there is some indication of a slight minimum in the curve at the edge of the Coulomb region.

NUCLEAR REACTIONS OF LOW-Z ELEMENTS WITH 5.7-Bev PROTONS. Paul A. Benioff,* Lawrence Radiation Laboratory, University of California, Berkeley, California (Received December 18, 1959).

This work describes the results of 5.7-Bev proton bombardments of the target elements Be, C, N, O, F, Na, and Al. Production cross sections were obtained for many radioactive products with half-lives between 1.2 minutes and 2.6 years. The (p,pn) cross sections for the targets C, N, O, F, and Na were found to be 29 ± 3 mb, 7.3 ± 0.7 mb, 33 ± 5 mb, 19 ± 2 mb, and 31 ± 5 mb, respectively. Much of the variation in these values is thought to be due to the difference in the number of neutrons available for (p,pn) reactions in the different target nuclei. The cross sections for other types of reactions studied do not change as much over the above range of target elements as do the (p,pn) cross sections. Comparison of the cross sections measured in this work with those obtained at 0.98 to 3 Bev shows that in the 1- to 5.7-Bev energy range the excitation functions are nearly constant.

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NUCLEAR STRUCTURE AND SIMPLE NUCLEAR REACTIONS. Paul A. Benioff,* Lawrence Radiation Laboratory, University of California, Berkeley, California (Received December 18, 1959; revised manuscript received March 9, 1960).

Recently it has become increasingly evident that some assumptions in the nuclear model used for the Monte Carlo calculations yield cross-section values which are not in accord with experiment. In particular, calculations of (p,pn)-reaction cross sections in the Bev energy range give values which are low by factors of two to nine when compared to experimental values. The calculated cross sections also show a smooth variation with the target atomic weight whereas the experimental values show quite an erratic variation. Reasons which have been advanced to account for this lack of agreement are the lack of a nuclear surface and failure to account for shell effects in the nuclear model used.

In this work a theory is developed to take account of surface and shell effects and thereby describe the observed magnitude and variation of the cross sections for simple nuclear reactions as exemplified by the (p,pn) reaction. At multi-Bev energies to which this treatment is restricted, the main contribution to the (p,pn)-reaction cross section comes from inelastic collisions between the incident protons and target neutrons, with all the p-n collision products escaping without further interaction. Approximations and assumptions used include the im-

pulse approximation, 0° lab scattering angle for the inelastic p-n collision products, classical trajectories for the incident and scattered particles, and a quantum-mechanical treatment for the target nucleons. The multi-Bev, n-p, cloud-chamber data were used to determine the average total exit cross section for the inelastically scattered particles. The only neutron shells in the target nucleus contributing to the (p,pn) reaction are those for which the instantaneous knocking out of a neutron creates a product-neutron hole state stable to particle emission. The combination of these assumptions gives integral expressions which, when evaluated on the IBM-701 computer for the independent-particle harmonic-oscillator shell model, give the (p,pn)-reaction cross sections as a function of the nuclear density distribution and the number of available shells.

For the low-Z nuclei where the available shells can be unambiguously determined, the results give a half-central-density radius parameter, r_0, ($r_0 = R_{1/2}/A^{1/3}$), of about 1.2 fermis compared to 1.03 fermis for the charge half-radius from the electron-scattering work. Use of reasonable limits on the value of r_0 allows one to set the minimum number of shells available for some targets. For example, the Zn^{64}, Cu^{65}, and Cu^{63} (p,pn) cross sections require that a large part or all the $1f_{7/2}$ neutrons be available, or, equivalently, that a $1f_{7/2}$ neutron hole state (across a major shell) in the product nucleus have less than 8- to 9-Mev excitation energy. The results also show that the energy associated with nuclear rearrangement to particle-stable product states must be less than 8 to 9 Mev. In several cases, the upper limit can be lowered considerably (to 1.5 Mev and 0 Mev in the cases of O^{16} and N^{14}, respectively).

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THEORY OF THE LOW-ENERGY PION-PION INTERACTION. Geoffrey F. Chew and Stanley Mandelstam, Lawrence Radiation Laboratory and Department of Physics, University of California, Berkeley, California (Received January 18, 1960).

The double-dispersion representation is applied to the problem of pion-pion scattering, and it is shown that, if inelastic effects are important only at very high energies and S-wave scattering dominates at low energy, a set of integral equations for the low-energy amplitudes can be derived. The solution of these equations depends on only one arbitrary real parameter, which may be defined as the pion-pion coupling constant. The order of magnitude of the new constant is established, and a procedure for solving the integral equations by iteration is outlined. If P-wave scattering is large the equations become singular and must be modified. Such a modification can be performed, at the expense of introducing an extra parameter, but is not considered here.

S-WAVE DOMINANT SOLUTIONS OF THE PION-PION INTEGRAL EQUATIONS. Geoffrey F. Chew, Stanley Mandelstam, and H. Pierre Noyes, Lawrence Radiation Laboratory and Department of Physics, University of California, Berkeley and Livermore, California (Received January 18, 1960).

The integral equations for pion-pion scattering formulated by Chew and Mandelstam are put into a form suitable for numerical solution. An iteration procedure is described that is applicable when the S-wave amplitude dominates the equations, all higher partial waves being small; this paper considers only solutions for which such is the case. The requirement that the equations have consistent solutions without bound states turns out to limit the pion-pion coupling constant to the range $-0.46 < \lambda \lesssim 0.3$. Results are given for various values of λ within this interval.

$|\Delta I| = \frac{1}{2}$ RULE AND THE WEAK FOUR-FERMION INTERACTION. S. Oneda* and J. C. Pati, University of Maryland, College Park, Maryland, and B. Sakita, University of Wisconsin, Madison, Wisconsin (Received February 5, 1960).

Although the usually considered diagram for the $\Lambda \to N + \pi$ decay arising from the interaction $(\bar{p}\Lambda)(\bar{n}p)$ can explain the decay rate, branching ratio, and asymmetry parameter of Λ decay, it fails to explain (a) the approximate validity of the $|\Delta I| = \frac{1}{2}$ rule, and (b) that the leptonic decay rates of the strange particles are slower than the universal rate, while the nonleptonic modes have nearly the universal rate. Introducing the effect of renormalization at the vertices of the strongly interacting particles phenomenologically, we have estimated the contributions to Λ decay

from a set of diagrams satisfying the strict $|\Delta I| = \frac{1}{2}$ rule for both local and nonlocal Fermi interactions. It is found that they are considerably more important than the usual diagram. This makes it easier to explain the approximate $|\Delta I| = \frac{1}{2}$ rule. Moreover, since these diagrams do not contribute to leptonic modes, one can understand (b) by associating the strangeness-nonconserving current with a weaker coupling constant. These important classes of diagrams lead to different restrictions on the chiralities of the currents involved in Λ decay for local and nonlocal interactions.

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BOSON FURRY THEOREM. D. C. Peaslee, Australian National University, Canberra, A.C.T., Australia, and M. T. Vaughn,* Purdue University, Lafayette, Indiana (Received October 12, 1959; revised manuscript received January 29, 1960).

A Furry theorem for heavy mesons and photons is given for a class of highly symmetric interactions, neglecting the Ξ-N mass difference. Because of this neglect, most rules are only approximately valid, but a few depend on charge conjugation alone and are absolute.

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ASYMPTOTIC CONDITIONS AND PERTURBATION THEORY. K. Nishijima,* Department of Physics, Osaka City University, Osaka, Japan and Department of Physics, University of Illinois, Urbana, Illinois (Received December 21, 1959).

The formulation of field theories based on few postulates but not using Lagrangian, Hamiltonian, or field equations has been investigated by many authors. In such a theory the coupled integral equations for Green's functions play the role of a substitute for field equations and serve to determine various physical quantities from the theory. In this paper we give a general prescription for the systematic solution of the coupled integral equations in perturbation theory.

We discuss two kinds of Green's functions: (1) the retarded functions, and (2) the time-ordered functions. Especially in the solution of the latter we have to use dispersion relations, and one finds a complete correspondence between the numbers of subtractions in the dispersion relations and the types of interactions in the conventional field theory; furthermore, the so-called renormalized coupling constants can be introduced into our theory through the boundary conditions supplementing the subtracted dispersion relations. On the contrary, however, it is not possible even to define the unrenormalized coupling constants in our scheme. In other words, all unobservable divergent quantities are completely eliminated in our formulation, and no divergences occur in the course of the entire calculations.

We apply this method to quantum electrodynamics to illustrate the above statement and also to show how one can get convergent unambiguous solutions in agreement with the conventional renormalized quantum electrodynamics. In discussing quantum electrodynamics, it is necessary to discover how to express the requirement of gauge invariance without referring to Lagrangian, Hamiltonian, or field equations; it is found that a set of equations which is a straightforward generalization of the Ward identity meets this requirement.

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CHARGED-SCALAR STRONG-COUPLING THEORY. H. Nickle* and R. Serber, Columbia University, New York, New York (Received February 4, 1960).

A treatment of the charged-scalar strong-coupling theory is given which employs a somewhat different choice of variables than that usually used; one which is more convenient for a discussion of the effects of quantum mechanical field fluctuations. The expansion parameter of the strong-coupling theory is shown to be $(1/g^2)\ln(1/Ka)$, where a is the source radius. The isobar energy is calculated to order $1/g^4$, and terms of order $(1/g^2)\ln(1/Ka)$ relative to the leading $1/g^2$ term are found to appear. Similar terms occur in the charge-renormalization factor. The logarithmic term in the isobar energy is found to be precisely that required to renormalize the charge; that is, the isobar energy, if expressed in terms of the renormalized coupling constant, is explicitly independent of the source radius.

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HIGH-ENERGY LIMIT OF FORM FACTORS. S. D. Drell and F. Zachariasen, Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California (Received February 2, 1960).

This theorem is proved: for finite charge renormalization constant Z_3^{-1}, the form factors describing any vertex with two particles on the mass shell must vanish at infinite momentum transfer. The relation of this result to the work of Lehmann, Symanzik, and Zimmermann is discussed.

COUPLED INTEGRAL EQUATIONS FOR THE NUCLEON AND PION ELECTROMAGNETIC FORM FACTORS. M. Baker and F. Zachariasen, Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California (Received February 2, 1960).

The dispersion relations for the nucleon isotopic vector form factors and the pion form factor which take into account contributions from both the 2π and NN intermediate states become a set of coupled integral equations for the form factors if the four amplitudes $(\pi\pi|NN)$ $(\pi\pi|\pi\pi)$ $(NN|\pi\pi)$ $(NN|NN)$ are assumed known. If these four amplitudes are replaced by their Born approximation values and spin and certain kinematic factors are neglected, the resulting set of coupled singular integral equations can be solved exactly. Comparison of these exact solutions with the form factors obtained from the usual approximation of retaining only the lowest mass state (i.e., the 2π state) confirms the hope that high-mass states do not contribute much to dispersion integrals. It is also of interest that these solutions are obtained from dispersion relations without subtractions and satisfy the necessary conditions that they vanish at infinite momentum transfer and take on the value e at the origin for all values of the coupling parameters appearing in the equations.

TIME-DEPENDENT IMPULSE APPROXIMATION. Saul T. Epstein, University of Nebraska, Lincoln, Nebraska (Received January 25, 1960).

The impulse approximation is generalized to cover cases in which a bound system is subject to a time-dependent perturbation. It is shown that the approximation is exact if the perturbation is an impulse. This result supports the supposition that the usual impulse approximation is accurate for collisions in which the collision time is short.